# Hybrid Approach to Design of Two Dimensional Stable IIR Digital Filter

Hichem CHAKER[(1)*], Samir KAMECHE[(2)]
[(1)] Department of Electronic. University Djillali Liabes of Sidi Bel Abbès, RCAM Laboratory, Algeria
[(2)] *Departement of Telecom, University of Tlemcen, Tlemcen,Algeria*
[*]hichem.chaker@univ-sba.dz

**Abstract:** In this paper, the magnitude optimization for the design of two dimensional digital filter is attempted and new design method for a two-dimensional (2-D) recursive digital filter is proposed via appropriate genetical swarm optimization with the introduction of the desired stability constraints. The method is tested on a numerical example and the obtained results are presented with previously published methods.

**Keywords:** Stable 2-Drecursive digital filter, Constrained Optimization, Genetical swarm optimization, Multidimensional Systems

## 1. INTRODUCTION

The design of 2-D (two-dimensional) filter has been investigated by several academic scholars and industry practitioners, due to the vast area of their applications in artificial vision, digital image processing, remote sensing, medical data processing, stereoscopy, radar and sonar data processing, geophysics astronomy, computer vision, pattern recognition attracted the attention [1,4]. Various methods for the design of 2-D recursive or non-recursive discrete signal have been proposed during the last decade by many authors. Also, due to the rapid progress from analog to digital systems, from analog to digital transmission and communication, there is a higher need for designing of 2-D discrete digital filters. An excellent survey is given in [5]. A great number of research works have been published in academic journals on the design of 2-D digital filters in the last 20 years. Design techniques for 2-D filters can be widely arranged into two categories: the first based on convenient transformation of 1-D filters [3,5] and the second one based on appropriate optimization techniques [5].

The heuristic methods have been employed to design the 2-D IIR filter, such as neural networks [6], genetic algorithm [7] and the computer language GENETICA [8]. These techniques were able to find out better solutions than those mentioned in the previous paragraph.

On the other hand, the stability of the designed filters is important for their practical implementation. In a recent publication [6], the authors suggested the magnitude optimization for the Design of 2-D Recursive Filters by using neural network considering appropriate constraints that ensure the 2-D filter stability. In this contribution, an attempt to design of 2-Dimensional stable recursive filters considering magnitude by using adaptive particle swarm optimization with genetic algorithm (GSO) has been made. A novel hybrid (GSO) algorithm, combining the forces of adaptive particle swarm optimization with genetic algorithm has been proposed. The hybrid approach combines the standard velocity and update rules of (APSO) with the notions of selection, crossover and mutation from genetic algorithm (GA). In the present paper numerical results show that the hybrid model methodology yields a better approximation to the transfer function, the proposed technique also satisfies the stability criterion which is introduced as constraints to the maximization problem. Conclusions are drawn in table 1.

## 2. PROBLEM FORMULATIONS

It is conventional when one considers 2-D stability problems, the notation of 2-D z-transformation with positive powers, i.e. for a two-dimensional sequence $f(n_1,n_2)$ ($f(n_1,n_2)=0$ for $n_1<0$ or $n_2<0$) to be used:

$$F(z_1,z_2)=\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty} f(n_1,n_2)z_1^{n_1}z_2^{n_2} \quad (1)$$

Consider $M_d$, the desirable magnitude response of a 2-D filter as a function of the frequencies $\omega_1$, $\omega_2$, ($\omega_1$, $\omega_2 \in [0,\pi]$). The design task is to find transfer function $H(z_1,z_2)$ such that the function $H(e^{j\omega_1},e^{j\omega_2})$ approximates the wished for magnitude response $M_d(\omega_1, \omega_2)$ as in [7].

For the design goal we consider that the realizable transfer function can be considered as:

$$H(z_1,z_2)=H_0\frac{\sum_{i=0}^{K}\sum_{j=0}^{K}a_{ij}z_1^i z_2^j}{\prod_{k=1}^{K}(1+b_k z_1+c_k z_2+d_k z_1 z_2)},\ a_{00}=1 \quad (2)$$

This transfer function can be simply implemented with software or hardware as a result of the ratio of two polynomials in the complex variables $z_1$, $z_2$ with real coefficients. This transfer function must perfectly approximate the desirable transfer function of the 2-D filter as regards the magnitude response. As one can see in (2) we utilize for the design aim separable or factorizable denominator into "first-order" polynomial factors, due to the presence of explicit stability requirements for each first order polynomial factor. Such definite explicit stability conditions have not been found for the general two-dimensional polynomial [1,4]. This approximation can be attained by minimizing (4).

$$J = J(a_{ij}, b_k, c_k, d_k, H_0)$$

$$= \int_{\omega_1=-\pi}^{\pi}\int_{\omega_2=-\pi}^{\pi}|M(\omega_1,\omega_2)-M_d(\omega_1,\omega_2)|^P\, d\omega_1 d\omega_2 \quad (3)$$

$P$ even positive integer (generally $p=2$ or $p=4$), $\omega_1=\frac{\pi}{N_1}n_1$ and $\omega_2=\frac{\pi}{N_2}n_2$.

$$\min J=\sum_{n_1=0}^{50}\sum_{n_2=0}^{50}\left[\left|M\left(\frac{\pi}{50}n_1,\frac{\pi}{50}n_2\right)\right|-M_d\left(\frac{\pi}{50}n_1,\frac{\pi}{50}n_2\right)\right]^2 \quad (4)$$

$$M(\omega_1,\omega_2)=\left|H(z_1,z_2)\big|_{\substack{z_1=e^{-j\omega_1}\\ z_2=e^{-j\omega_2}}}\right| \quad (5)$$

We have to minimize the functional (4), the aim is to minimize the difference between current and desired magnitude response of the 2-D filter in $N_1\times N_2$ points. Since the denominator only contains first-degree factors, we can asseverate the stability conditions as the constraints [1,3].

It is renowned also that a linear, causal, shift-invariant, single-input, single-output, discrete variables 2-D system is BIBO (Bounded Input Bounded Output) stable if and only if its output is bounded when the input is bounded and the initial conditions are zero. For a 2-D filter, with transfer function $H(z_1,z_2)=A(z_1,z_2)/B(z_1,z_2)$, $A(z_1,z_2)$ and $B(z_1,z_2)$ coprime polynomials in $z_1$ and $z_2$, Huang's Theorem guarantees Bounded Input Bounded Output Stability if and only if the following two conditions are satisfied [1,4]:

a) $B(0,z_2)\neq 0$, for $|z_2|\leq 1$ and

b) $B(z_1,z_2)\neq 0$, for $|z_1|\leq 1$, $|z_2|=1$

For 2-D filter design purposes, we need more practical relations. So from the 2-D systems theory if $B(z_1,z_2)$ is a product of low order polynomials as in (2) we do have such relations. Thus, if $B(z_1,z_2)$ is as in (2), we have the more applicable necessary and stability conditions:

$$|b_k+c_k|-1<d_k<1-|b_k-c_k| \quad ,k=1,2,...K \quad (6)$$

Thereby the design of the two dimensions recursive filters is equivalent to the following constrained minimization problem:

$$\min J=\sum_{n_1=0}^{50}\sum_{n_2=0}^{50}\left[\left|M\left(\frac{\pi}{50}n_1,\frac{\pi}{50}n_2\right)\right|-M_d\left(\frac{\pi}{50}n_1,\frac{\pi}{50}n_2\right)\right]^2 \quad (7)$$

Subject to:

$$\begin{cases}|b_k+c_k|-1<d_k\\ d_k<1-|b_k-c_k|\end{cases} \quad ,k=1,2,...K \quad (8)$$

We consider the case K=2 and $H(z_1,z_2)$ from (2) is:

$$H(z_1,z_2)\big|_{\substack{z_1=e^{-j\omega_1}\\ z_1=e^{-j\omega_1}}}=H_0\frac{A_R-jA_I}{(B_{1R}-jB_{1I})(B_{2R}-jB_{2I})} \quad (9)$$

Where:

$$A_R=a_{00}+a_{01}c_{01}+a_{02}c_{02}+a_{10}c_{10}+a_{20}c_{20}+a_{11}c_{11}+a_{12}c_{12}+a_{21}c_{21}+a_{22}c_{22}$$

$$A_I=a_{01}s_{01}+a_{02}s_{02}+a_{10}s_{10}+a_{20}s_{20}+a_{11}s_{11}+a_{12}s_{12}+a_{21}s_{21}+a_{22}s_{22}$$

$$B_{1R}=1+b_1c_{10}+c_1c_{01}+d_1c_{11}$$

$$B_{1I}=b_1s_{10}+c_1s_{01}+d_1s_{11}$$

$$B_{2R}=1+b_2c_{10}+c_2c_{01}+d_2c_{11}$$

$$B_{2I}=b_2s_{10}+c_2s_{01}+d_2s_{11}$$

With

$$c_{\xi_1,\xi_2}=c_{\xi_1,\xi_2}(\omega_1,\omega_2)=\cos(\xi_1\omega_1+\xi_2\omega_2),$$
$$\xi_1,\xi_2=0,1,2$$

$$s_{\xi_1,\xi_2}=s_{\xi_1,\xi_2}(\omega_1,\omega_2)=\sin(\xi_1\omega_1+\xi_2\omega_2),$$
$$\xi_1,\xi_2=0,1,2$$

Therefore we obtain for $M(\omega_1,\omega_2)$:

$$M(\omega_1,\omega_2)=H_0\frac{\sqrt{A_R^2+A_I^2}}{\sqrt{B_{1R}^2+B_{1I}^2}\sqrt{B_{2R}^2+B_{2I}^2}} \quad (10)$$

This problem has been tackled using GAs [7]. In this paper, we give a solution using GSO. We will show the present method yields a better solution. Furthermore, we proceed with the design of the 2-D recursive filter given by (1) for the case K=2. Let the desired amplitude response be given by:

$$M_d(\omega_1,\omega_2)=\begin{cases}1 & if \quad \sqrt{\omega_1^2+\omega_2^2}\le 0.08\pi \\ 0.5 & if \quad 0.08\pi<\sqrt{\omega_1^2+\omega_2^2}<0.12\pi \\ 0 & otherwise\end{cases} \qquad (11)$$

### 3. GENETICAL SWARM OPTIMIZATION

A new hybridization of the two well-known evolutionary adaptive particle swarm optimization (APSO) [14,17] and genetic algorithm (GA) [9,13]] methodologies is presented and the infrastructure is showed in this section. The applied approach merges the evolutionary natures of the (APSO) and (GA) operate with the same initial generation and the individuals are randomly generated. These particles or individuals may be considered as a particle in the case of the adaptive particle swarm optimization, or as a chromosome in the case of genetic algorithm. In the (GSO) [18,20], to create the new generation we incorporate the selection, crossover and mutation concepts from genetic algorithm, and social and cultural rules from the adaptive particle swarm optimization. The initial generation is divided into two sub-population and they arethen considered again in a new modified population that is anew randomly splitted in two parts in the upcoming iteration cycle for a further execution of (APSO) or (GA) procedure.

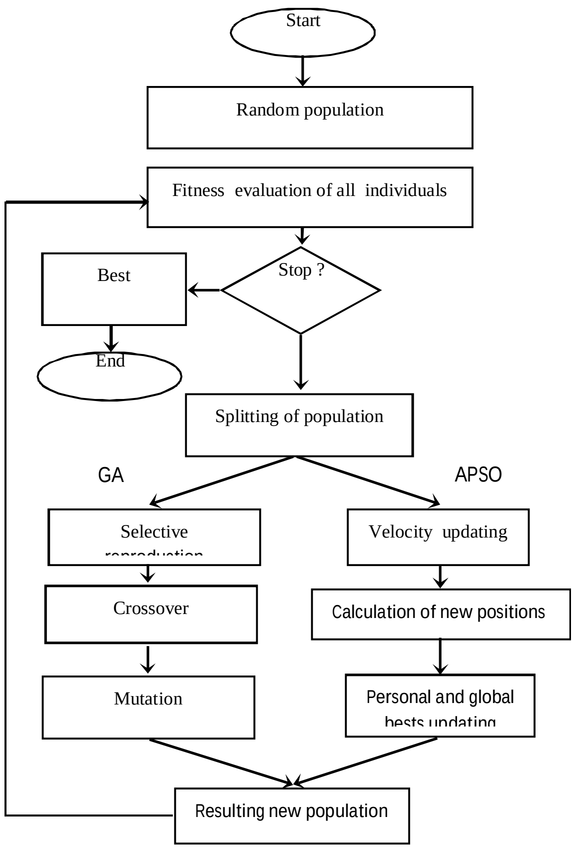


Fig. 1 Flow-chart of the GSO algorithm

The genetical swarm optimizer parameters are as follows: number of particles equal to 40, inertia weight $w$ from 0.9 to 0.4, $C_1$ and $C_2$equal to 0.5, $T_c$=3, $P_m$=0.1, $P_c$=0.4 and HC= 0.5.

Table 1 Parameter configuration for the (GSO) algorithm

| Algorithms | Parameters | Value |
|---|---|---|
| APSO | Population size | 40 |
| | Inertiaweight$w$ | 0.9 to 0.4 |
| | $C_1$ , $C_2$ | 0.57 |
| | $V_{max}$ | 3 |
| | $T_c$ | 3 |
| GA | $P_m$ | 0.1 |
| | $P_c$ | 0.4 |
| GSO | HC | 0.5 |

The (GSO) procedure is guided by the Hybridization Coefficient (HC), it designates the corresponding percentage of population that is processed with (APSO) or (GA). HC equal to 0.5 that means that 20 particles is evolved with (APSO) procedure, while the remained one are with the (GA) approach.

## 4. NUMERICAL RESULTS

The coordinates of the particles were initialized with a random numbers whose absolute values were kept below 3. The maximum admissible velocity $V_{max}$ for each particle was restricted to the upper value of the dynamic distance of search, i.e., $|V_{max}| = |X_{imax}| = 3$. We need to represent each trial solution as a particle in a multi-dimensional search interval. Since $a_{00}$ is always set to 1 in (2), the dimensionality of the introduced present problem is 14 and each particle has 14 positional coordinates symbolized by the vector $X(a_{00}, a_{01}, a_{02}, a_{11}, a_{12}, a_{20}, a_{21}, a_{22}, b_1, b_2, c_1, c_2, d_1, d_2, H_0)^T$ ,In this application, we have calculated J by setting $N_1 = N_2 = 50$ in (4). For the search procedure we have employ a population with a dimension of 40. The best result is achieved for p=2 in 3301 computational cycles. Result of the aforesaid search methodology is depicted in Table 2 with results given by other approaches. Figure 2 shows the graphic representation of the desired amplitude response (11) in the range [0,π]× [0,π], while figure 3 shows the graphic representation of (10) resulting from the coefficient values given in the Table 2.

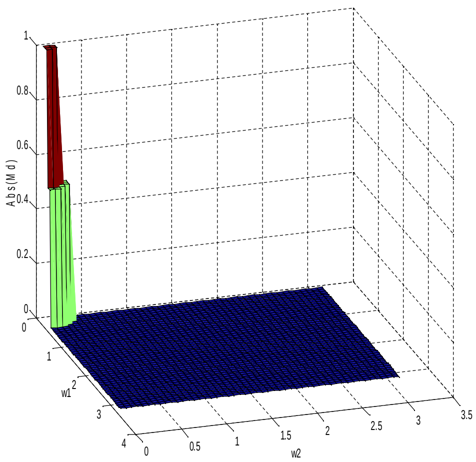

Fig. 2 Desired amplitude response $|M_d(\omega_1,\omega_2)|$ of 2-D filter

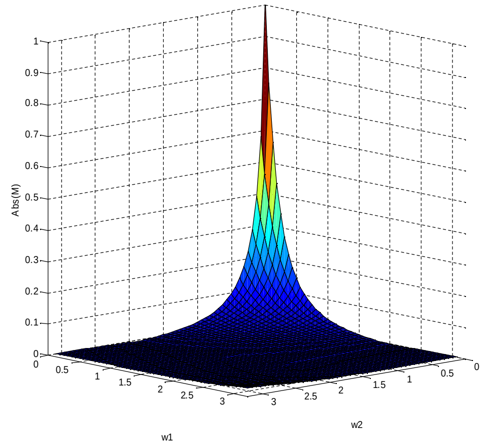

Fig. 3 Amplitude response $|M(\omega_1,\omega_2)|$ of the considered 2-D filter using GSO

Table 2 Best values of filter coefficients generated by GSO and the competitor algorithm

| | GSO | Results of [5] | Results of [6] | Results of [7] |
|---|---|---|---|---|
| $a_{01}$ | 1.3661 | 1.8922 | 1.8162 | 0.3233 |
| $a_{02}$ | 0.6216 | -1.2154 | -1.1060 | 1.1108 |
| $a_{10}$ | 1.0276 | 0.0387 | 0.0712 | 0.3899 |
| $a_{11}$ | 1.2800 | -2.5298 | -2.5132 | -0.0437 |
| $a_{12}$ | 1.5862 | 0.3879 | 0.4279 | 0.6007 |
| $a_{20}$ | 2.5017 | 0.6115 | 0.5926 | 1.0776 |
| $a_{21}$ | 0.5472 | -1.4619 | -1.3690 | 0.5095 |
| $a_{22}$ | 1.6216 | 2.5206 | 2.4326 | 0.4758 |
| $b_1$ | -0.3287 | -0.8707 | -0.8662 | -0.9697 |
| $b_2$ | 0.2622 | -0.8729 | - 0.8907 | 0.0031 |
| $c_1$ | 0.0660 | -0.8705 | - 0.8531 | -0.9681 |
| $c_2$ | -0.4849 | -0.8732 | - 0.8388 | -0.0225 |
| $d_1$ | -0.4263 | 0.7756 | 0.7346 | 0.9521 |
| $d_2$ | -0.7586 | 0.7799 | 0.8025 | -0.9162 |
| $H_0$ | 0.0032 | 0.0010 | 0.0009 | 0.0003 |

## 5. CONCLUSION

In this paper a new optimization methodology GSO has been utilized to solve the problem of designing 2-D digital stable recursive filters. The obtained filter has a very good stability margin. The applied procedure leads to a simpler filter since in practice we have to produce a factorable denominator. The genetical swarm optimization algorithm applied here yields a good design.

***References***


[1] T. Kaczorek,Two-Dimensional Linear Systems, Springer-Verlag, Berlin- Heidelberg, 1985.

[2]B. A. Shenoi, Magnitude and Delay Approximation of 1-D and 2-D Digital Filters, Springer Verlag. Berlin, Heidelberg, New York, 1999.

[3] H. R Adardour, S. Kameche, "Analysing Primary Signal Sensing Test In Cognitive Radio Networks Using An Alpha-beta Filter And A Neyman-pearson Detector," Algerian Journal of Signals and Systems Vol 3, Issue 2, pp 70-80, december 2018.

[4]Abderrahmane Ouadi, Hamid Bentarzi and Abdelmadjid Recioui, Optimal multiobjective design ofdigital filters using spiral optimization technique, SpringerPlus, September 2013, 2:461.

[5] W.S. Lu and A. Antoniou, Two-Dimensional Digital Filters, Marcel Dekker. New York, 1992.

[6] V.M. Mladenov and N. E. Mastorakis, "Design of two-dimensional digital recursive filters by using neural networks", IEEE Trans. Neural Netw,vol.12, Issue:3, May 2001.

[7]N. E. Mastorakis, I. F. Gonos, and M. N. S. Swamy, "Design of two dimensional recursive filters using genetic algorithms", IEEE Trans.Circuits Syst. I, Fundam. Theory Appl, vol. 50, Issue:5, May2003.

[8] I. F. Gonos, L. I. Virirakis, N. E. Mastorakis, and M. N. S. Swamy, " Evolutionary design of 2-dimensional recursive filters via the computer language GENETICA", IEEE Trans. Circuits Syst. II, vol.53, Issue:4, April 2006.

[9] D. E. Goldberg, Genetic Algorithms in Search, Optimization and Machine Learning . Addison-Wesley, New York, 1989.

[10] Y. Rahmat-Samii and E. Michielssen, Electromagnetic Optimization by Genetic Algorithms. Wiley, New York, 1999.

[11] J. Arabas, Z. Michalewicz and J. Mulawka.: "GA Vs PS – A Genetic Algorithm with Varying Population Size", [Proceedings of IEEE International Conference on Evolutionary Computation, p.73, 1994].

[12] R. Tanese, "Distributed Genetic Algorithm", [Proceedings of the International Conference on Genetic Algorithms, 1989].

[13] S. M. Elsayed, R. A. Sarker and D. L. Essam, "Improved genetic algorithm for constrained optimization," [Proceedings of the International Conference on Computer Engineering & Systems, p.111, 2011].

[14] J. Kennedy, "The particle swarm: social adaptation of knowledge", [ IEEE Int. Conf. on Evolutionary Computation, p.303, 1997].

[15] R. Eberhart and Y. Shi, "Particle swarm optimization: developments, applications and resources", [IEEE Int. Conf. on Evolutionary Computation, p.81, 2001].

[16] G. Nicolis and I. Prigogine, Self-organization in no equilibrium systems: from dissipative systems to order through fluctuations. John Wiley, New York, 1977.

[17] X. Xiao-Feng, Z. Wen-Jun and Y. Zhi-Lian, "Adaptive Particle Swarm Optimization on Individual Level", [International Conference on Signal Processing, p.1215, 2002].

[18] D. W. Boeringer and D. H. Werner, "Particle Swarm Optimization Versus Genetic Algorithms for Phased Array Synthesis", IEEE Transactions on Antennas and Propagation,vol. 52, Issue:3, March 2004.

[19] J. Robinson, S. Sinton and Y. Rahmat Samii, "Particle Swarm, Genetic Algorithm, and Their :Hybrids: Optimization of a Profiled Corrugated Horn Antenna", [IEEE Proceedings of the International Symposium on Antennas and Propagation, p.314, 2002].

[20] J. Chia-Feng, "A Hybrid of Genetic Algorithm and Particle Swarm Optimization for Recurrent Network Design", IEEE Transactions on Systems, Man, And Cybernetics-Part B: Cybernetics, vol. 34, Issue:2, April 2004.