A Novel Fractionalized PID controller Using The Sub-optimal Approximation of FOTF

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Abstract: In the last two decades, fractional calculus has been rediscovered by scientists and engineers and applied in an increasing number of fields, namely in the area of control theory. Recently, many research works have focused on fractional order control (FOC) and fractional systems. It has proven to be a good mean for improving the plant dynamics with respect to response time and disturbance rejection. In This work we use the Sub-optimal Approximation of fractional order transfer function to design the parameters of PID controller and we study the performance analysis of fractionalized PID controller over integer order PID controller.

Keywords: Fractional Control, Approximation Methods, Oustaloup Method, PID Controller.

1. INTRODUCTION

The theory of fractional-order derivative was developed mainly in the 19th century. Recent books [1],[2],[3] provide a good source of references on fractional calculus. However, applying fractional-order calculus to dynamic systems control is just a recent focus of interest [1],[4],[9].

The earliest theoretical contributions to the field were made by Euler and Lagrange in the eighteenth century, and the first systematic studies seem to have been made at the beginning and middle of the nineteenth century by Liouville, Riemann, and Holmgren. It was Liouville who expanded functions in series of exponentials and defined the nth-order derivative of such a series by operating term-by-term as though n were a positive integer. Riemann proposed a different definition that involved a definite integral and was applicable to power series with non-integer exponents. It was Grünwald and Krug who first unified the results of Liouville and Riemann. Grünwald, by returning to the original sources and adopting them as starting points. The definition of a derivative as the limit of a difference quotient and arriving at definite-integral formulas for the nth-order derivative. Krug, working through Cauchy's integral formula for ordinary derivatives, showed that Riemann's definite integral had to be interpreted as having a finite lower limit while Liouville's definition corresponded to a lower limit $-\infty$.

The first application of the fractional calculus was made by Abel in 1823. He discovered that the solution of the integral equation for the tautochrone problem could be obtained via an integral in the form of derivative of order one half. Later in the nineteenth century, important stimuli to the use of fractional calculus were provided by the development by Boole of symbolic methods for solving linear differential equations of constant coefficients, or the operational calculus of Heaveside developed to solve certain problems in electromagnetic theory such as transmission lines.

In the twentieth century contributions have been made to both the theory and applications of fractional calculus by very well known scientists such as Weyl and Hardy (properties of difier integrals), Erdely (integral equations), Riesz (functions of more than one variable), Scott Blair (rheology), or Oldham and Spanier (electrochemistry and general transport problems).

In the last decades of the last century there was continuing growth of the applications of fractional calculus mainly promoted by the engineering applications in the fields of feedback control, systems theory, and signals processing However, this also implies that the tuning of the controller can be much more complex. In order to address this problem, different methods for the design of a FOPID controller have been proposed in the literature.

The concept of FOPID controllers was proposed by Podlubny in 1997. He also demonstrated the better response of this type of controller, in comparison with the classical PID controller, when used for the control of

fractional order systems.
The main contribution of this work is the use the fractionalized PID controller approach to reduce noise effect by introducing fractional order filters in the classical feedback control loop PID controller without changing the overall equivalent closed loop transfer function.
This paper is structured as follows: Section 2 is an fundamentals of fractional calculus. Section 3 presents numerical algorithm for Sub-optimal Rational Approximations, and the fractionalized PID controller and simulation results are given is given in section 4 and 5. Finally, the conclusion with future work are presented in section 6.

2. FUNDAMENTALS OF FRACTIONAL CALCULUS

Fractional calculus is an old mathematical topic since 17th century. Fractional calculus is a subdivision of calculus theory which generalizes the derivative or integral of a function to non-integer order. Fractional calculus helps evaluating (dny/dtn), n-fold integrals where n is fractional, irrational or complex. For fractional order systems n is considered to be fractional. The number of applications where fractional calculus has been used rapidly grows. These mathematical phenomena allow to describe a real object more accurately than the classical ―integer-order‖ methods. The real objects are generally fractional however, for many of them the fractionality is very low. The main reason for using the integer-order models was the absence of solution methods for fractional differential equations. At present there are lots of methods for approximation of fractional derivative and integral and fractional calculus can be easily used in wide areas of applications (e.g.: control theory - new fractional controllers and system models, electrical circuits theory-fractances, capacitor theory, etc.) [5],[6],[10].
The generalized fundamental operator which includes the differentiation and integration is given as [11],[12]:

$$ {}_aD_t^q = \begin{cases} \frac{d^q}{dt^q} & , R(q) > 0 \\ 1 & , R(q) = 0 \\ \int_a^t (d\tau)^{-q} & , R(q) < 0 \end{cases} \qquad (1) $$

where a_i and b_j are real numbers such that

$$ \begin{cases} 0 \le \alpha_0 \le \alpha_1 \le \cdots \le \alpha_n \\ 0 \le \beta_0 \le \beta_1 \le \cdots \le \beta_m \end{cases} $$

and s is the Laplace operator.

3. RATIONAL APPROXIMATIONS TO FRACTIONAL INTEGRATORS AND DIFFERENTIATORS

A. OUTSTALOUP'S METHOD

The approximation of Oustaloup a generalized derivator, differential action which covers the frequency space, based on a recursive distribution of an infinite number of zeros and negative real poles (to ensure phase behavior minimum). As part of a realist synthesis (practice) based on a finite number of zeros and poles, it should reduce the differential behavior of a generalized bounded frequency range, chosen according to the needs of the application [5],[7],[15].
The method is based on the function approximation from:

$$ H(s) = S^{\alpha} \quad , \alpha \in R^{+} \qquad (2) $$

By a rational function [1,3] :

$$ G_f(s) = K \prod_{k=1}^{N} \frac{s+w_k'}{s+w_k} \qquad (3) $$

Where the poles, zeros, and gain are evaluated from:

$$ w_k' = w_b . w_u^{\frac{2k-1-\gamma}{N}} $$
$$ w_k = w_b . w_u^{(2k-1+\gamma)/N} \;, K = w_h^{\gamma} $$

Where w_u is the unity frequencies gain and the central frequency of a band of frequencies distributed geometrically. Let $w_u = \sqrt{w_h w_b}$, where w_h and w_b are respectively the upper and lower frequencies.
γ is the order of derivative, and N is the order of the filter.

4. NUMERICAL ALGORITHM FOR SUB-OPTIMAL RATIONAL APPROXIMATIONS

Our target now is to find an approximate integer-order model with a relative low order, possibly with a time delay in the following form [3],[13],[16]:

$$ G_{\frac{r}{m},\tau}(s) = \frac{\beta_1 s^r + \cdots + \beta_r s + \beta_{r+1}}{s^m + \alpha_1 s^{m-1} + \cdots + \alpha_{m-1} s + \alpha_m} e^{-\tau s} \qquad (4) $$

An objective function for minimizing the H2-norm of the reduction error signal e(t) can be defined as:

$$J = \min_{\theta} \left\| \hat{G}(s) - G_{r/m,\tau}(s) \right\|_2$$

Where θ is the set of parameters to be optimized such that

$\theta = [\beta_1, \ldots, \beta_r, \alpha_1, \ldots, \alpha_m, \tau]$.

For an easy evaluation of the criterion J, the delayed term in the reduced order model Gr/m,τ (s) can be further approximated by a rational function Gr/m(s) using the Padé approximation technique [1],[4]. Thus, the revised criterion can then be defined by

$$J = \min_{\theta} \left\| \hat{G}(s) - \hat{G}_{r/m}(s) \right\|_2$$

And the H2 norm computation can be evaluated recursively using the algorithm in [1],[12],[13].

Suppose that for a stable transfer function type

$$E(s) = \hat{G}(s) - \hat{G}_{\frac{r}{m}}(s) = \frac{B(s)}{A(s)}$$

the polynomials Ak(s) and Bk(s) can be defined such that

$$A_k(s) = a_0^k + a_1^k s + \cdots + a_k^k s^k, \quad B_k(s) = b_0^k + b_1^k s + \cdots + b_{k-1}^k s^{k-1}$$

The values of a_i^{k-1} and b_i^{k-1} can be evaluated recursively from

$$a_i^{k-1} = \begin{cases} a_{i+1}^k & i \text{ iven} \\ a_{i+1}^k - \alpha_k\, a_{i+2}^k, & i \text{ odd} \end{cases}$$
$$i = 0, \ldots, k-1$$

and

$$b_i^{k-1} = \begin{cases} b_{i+1}^k & i \text{ iven} \\ b_{i+1}^k - \beta_k\, a_{i+2}^k, & i \text{ odd} \end{cases}$$
$$i = 0, \ldots, k-1$$

Where $\alpha_k = a_0^k / a_1^k$ and $\beta_k = b_1^k / a_1^k$.

The H2-norm of the approximate reduction error signal ê(t) can be evaluated from

$$J = \sum_{k=1}^{n} \frac{\beta_k^2}{2\alpha_k} = \sum_{k=1}^{n} \frac{(b_1^k)^2}{2\, a_0^k\, a_1^k} \qquad (5)$$

The sub-optimal H2-norm reduced order model for the original high-order fractional-order model can be obtained using the following procedure [1]:

1. Select an initial reduced model $\hat{G}^0_{r/m}(s)$
2. Evaluate an error $\left\| \hat{G}(s) - \hat{G}^0_{r/m}(s) \right\|_2$ from (5).
3. Use an optimization algorithm (for instance, Powell's algorithm) to iterate one step for a better estimated model $\hat{G}^1_{r/m}(s)$.
4. Set $\hat{G}^0_{r/m}(s) \longleftarrow \hat{G}^1_{r/m}(s)$, go to Step 2 until an optimal reduced model $\hat{G}^*_{r/m}(s)$ is obtained.
5. Extract the delay from $\hat{G}^*_{r/m}(s)$, if any.

5. THE FRACTIONALIZED ORDER PID CONTROLLER

The feedback control loop with an integer order controller is shown in figure 1 as [8],:

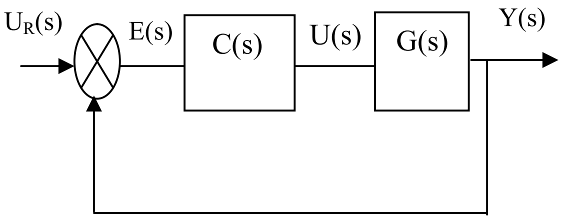

Fig. 1 The feedback control loop with an integer order controller

Where, $U_R(s)$: Input Signal
E(s) : Error Signal
C(s) : Controller Transfer Function
G(s) : System or plant Transfer Function
Y(s) : output Signal U(s) – Controller Signal

The integer-order PID controller to be designed is in the following form:

$$C(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s\right) \qquad (6)$$

The PID control scheme is modified here to get more robustness against noise and perturbation. The new PID control law is obtained by using the fractionalization of a control system element [14], the integral operator 1/s is fractionalized as represented in Figure 3 , that is,

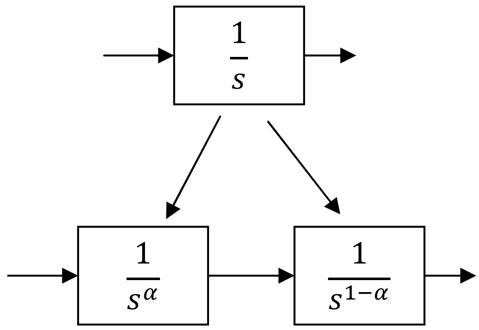

Fig. 2 Fractionalization of integral operator.

$$\frac{1}{s} = \frac{1}{s^{\alpha}}\frac{1}{s^{(1-\alpha)}}$$

where α is a real number such that $0 < \alpha < 1$.

The feedback control loop with an Fractionalized order controller is shown in figure 3 as:

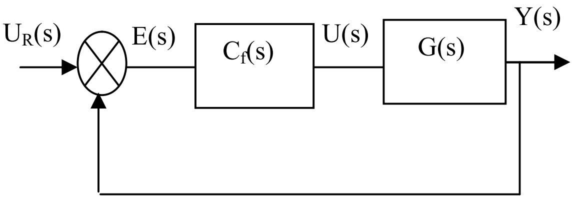

Fig. 3 The feedback control loop with an Fractionalized order controller

Where, $C_f(s)$: Fractionalized Controller Transfer Function

The Fractionalized of the integer-order PID controller to be designed is in the following form [13],[14]:

$$C_f(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

$$C_f(s) = \frac{1}{s}\left(\frac{(k_p T_d s^2 + k_p T_i s + k_p}{T_i}\right)$$

$$C_f(s) = \frac{1}{s^{\alpha}}\frac{1}{s^{(1-\alpha)}}\left(\frac{(k_p T_d s^2 + k_p T_i s + k_p}{T_i}\right) \quad (7)$$

were, $0 < \alpha < 1$

6. SIMULATION RESULTS AND DISCUSSION

Let us consider the following FO-LTI plant model :

$$G(s) = \frac{1}{s^{2.3} + 3.2s^{1.4} + 2.4s^{0.9} + 1} \quad (8)$$

Let us first approximate it with Oustaloup's method and then fit it with a fixed model structure known as first-order lag plus dead time (FOLPD) model [12],[16] , where

$$G_r = \frac{K}{Ts+1}e^{-Ls} \quad (9)$$

can perform this task and the optimal FOLPD model obtained is given as follows:

$$G_r(s) = \frac{0.9951}{3.5014\,S + 1}e^{-1.634\,S} \quad (10)$$

The comparison of the open-loop step response is shown in Figure 4. It can be observed that the approximation is fairly effective.

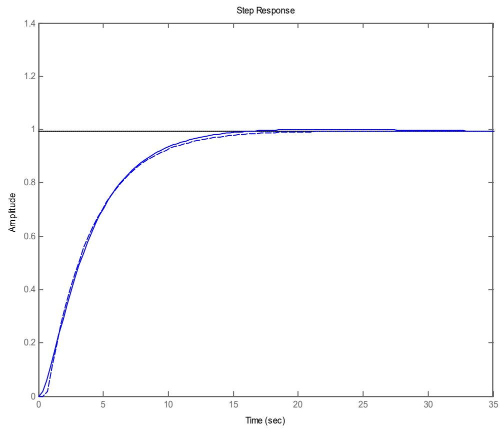

Fig. 4 Step response comparison of the optimum FOLPD and the original model

Designing a suitable feedback controller for the original FO-LTI system G can be a formidable task. Now let us consider designing an integer-order PID controller for the optimally reduced model Gr(s) and let us see if the designed controller still works for the original system.

The integer-order PID controller to be designed is in the following form [12],[13],[16]:

$$C(s) = K_p\left(1 + \frac{1}{T_i s} + \frac{T_d s}{\frac{T_d}{N}s + 1}\right)$$

The optimum ITAE criterion-based PID tuning formula [4] can be used:

$$K_p = \frac{\left(0.7303 + \frac{0.5307\,T}{L}\right)(T + 0.5\,L)}{K(T + L)} \quad (11)$$

$$T_i = T + 0.5\,L\,,\; T_d = \frac{0.5\,L\,T}{T + 0.5\,L}$$

The parameters of the PID controller are then Kp =3.4160,Ti =3.8164,Td =0.2890, and the PID controller can be written as

$$C(s) = \frac{1.086s^2 + 3.442\,s + 0.8951}{0.0289\,s^2 + s} \quad (12)$$

The parameters of the Fractionalized PID controller are then Kp =3.4160,Ti =3.8164,Td =0.2890, α=0.3 and the fractionalized PID controller can be written as:

$$C_f(s) = \frac{1}{s^{\alpha}}\frac{1}{s^{(1-\alpha)}}\frac{(1.086s^2 + 3.442\,s + 0.8951)}{(0.0289\,s + 1)} \quad (13)$$

$$= \frac{1}{s^{0.3}}\frac{1}{s^{0.7}}\frac{(1.086s^2 + 3.442\,s + 0.8951)}{(0.0289\,s + 1)}$$

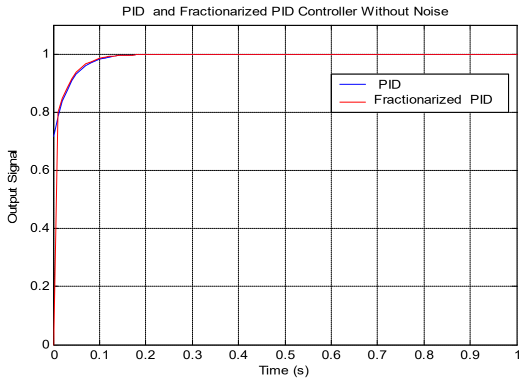

Fig. 5 Closed-loop step response of the fractionalized PID and the integer-order PID controller

Finally, the step response of the original FO-LTI with the above designed PID controller is shown in Figure 3. A satisfactory performance can be clearly observed. Therefore, we believe the method presented can be used for integer-order controller design for general FO-LTI systems.

Figure (6) shows the time response characteristics of the PID and fractionalized order PID controllers with random output noise of 3 % of the reference signal amplitude (α=0.5).

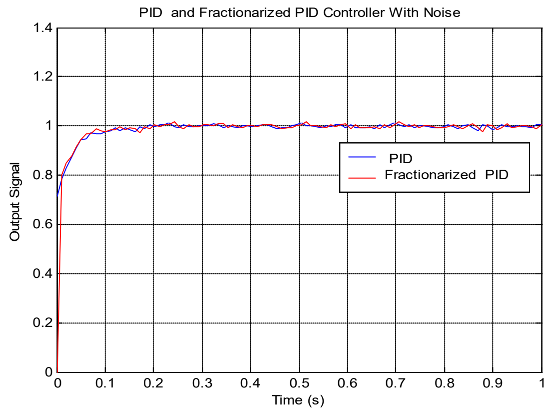

Fig. 6 The PID and fractionalized order PID controllers with random output noise of 3 % of the reference signal amplitude (α=0.5).

The PID and fractionalized PID controllers with random output noise of 20 % of the reference signal amplitude (α=0.5) is given in the following figure:

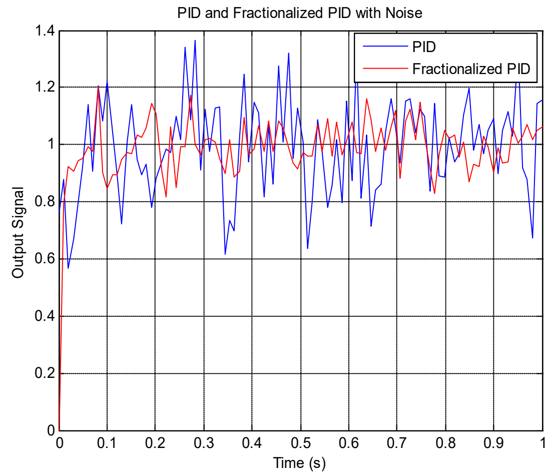

Fig. 7 The PID and fractionalized PID controllers with random output noise of 20 % of the reference signal amplitude (α=0.5).

7. ROBUSTNESS ANALYSIS

The evaluation of the control system performance will be realized by dening a quadratic error criterion J given by,

$$J_\alpha = \int_{t_I}^{t_F} (U_R(t) - Y(t))^2 dt \qquad (14)$$

The Quadratic error criterion with random output noise of 3% and 20% are given in Table1 and Table 2 respectively:

TABLE 1 - Quadratic error criterion with random output noise of 5 %

α	0.1	0.2	0.3	0.4	0.5	1
J	0.046	0.05	0.054	0.053	0.058	0.065

Table 2: Quadratic error criterion with random output noise of 20%

α	0.1	0.2	0.3	0.4	0.5	1
J	0.12	0.14	0.16	0.17	0.15	0.52

We remark that the fractionalized PID give the A certain diminution of the noise effect of about 50% comparatively to the classical PID result

8. CONCLUSION

In present work, we propose a new approach for PID robust control by introducing fractional order filters in the classical feedback control loop without changing the overall equivalent closed loop transfer function.

Based on a simple idea to introduce fractional order operators in the control system loop, the

Fractionalization approach allows to improve the noises rejection and the robustness of the control scheme.

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