The impact of fractional order control on multivariable systems

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Abstract: In this work a fractional order filter proportional integer (FOF-PI) controller is proposed for the control of multivariable system which is a distillation column model. In addition to the design of fractional controller, a comparison between integer controller employing the most used method the biggest log modulus (BLT) and the internal model control (IMC) with the fractional method. The purpose of this comparison is to improve the quality control and show the superiority and the increase of flexibility by the additional fractional parameter of the fractional control. An analyzed study between performance indices of the three methods as the overshoot, settling time, integral absolute error (IAE) and rise time is exhibited. The simulation graphs of the outputs provided by the three used controllers are presented to confirm the results and show the improvement in the control of multivariable system domain by the fractional method.

Keywords: BLT Method; IMC Method; PID; Fractional Controller; Distillation Column; Multivariable System; Fractional Filter; RGA; Interaction.

1. INTRODUCTION

Due to the importance of multivariable system in industries, it gets attention more and more. The biggest challenge in the control of multivariable systems is the complexity and the existence of interaction. In spite of considerable advanced multivariable system control, The PID controllers was and still dominate in industries control due to the simplicity in structure and implementation. Many different approaches are studied in purpose of tuning the PID parameters. From these techniques: Ziegler-Nichols tuning which proposed in 1942 [1], the methods based on the dominant pole placement [2] without forgetting the well-known method the biggest log modulus tuning (BLT) proposed by Luyben [3] [4]. This method provides reasonable parameter setting with guaranteed closed loop stability. In the same year 1986 appeared another well-known method which is the internal model control (IMC) by Economu and Morari [5]. The IMC method became a base for several other methods for the easy design [6].

For the improvement of PID controllers, it was developed a novel method of control which is the fractional control. The fractional control was occurred for the first time in 1999 by Podlubny [7]. FO-PID is the extension of PID controller with two additional elements to tune. In the last decades, it has been an attractive domain of researchers because of the interesting gotten results. In literature there is many studies for fractional control for SISO systems [8] and also an implementation simulation for industrial process [9] using System Identificationtoolbox in Matlab environment. Then it extended to the control of multivariable based on the advanced strategies like the use the of fractional fuzzy PID [10], predictive fractional PI [11], adaptive fractional order PID [12], PSO algorithm [13] and another method based on the IMC is discussed in [14].

In this paper, a comparative study for multivariable system control is presented be-tween integer controllers which present here by the most used methods the BLT and the IMC and in the other hand the fractional control method. The fractional controller used in this work and proposed by [14] has an interesting structure: integer part which is a simple PID controller and a fractional filter part.

2. METHODS

background

Before dealing with the method used in this paper, it is preferred to provide some notion about multivariable systems and the analyses of interaction due to its importance.

Multivariable systems are the systems which have no less than two-input two-output or more. It characterized by the existence of interaction which is the biggest obstacle for the control. The most used method to analyze the interaction is the relative gain array (RGA) method, that elaborated by Bristol in 1966 [15]. It enables a less interactive control configuration and simple to interpret. The relative gain array is a matrix n*n (dimension of the system), based on the static gain matrix of the open loop sys-tem. Each element of the RGA matrix λ_{ij} expressed by the following equation:

$$\lambda_{ij} = \frac{\left(\frac{\partial y_i}{\partial u_j}\right)_{u_{k=0;k\neq j}}}{\left(\frac{\partial y_i}{\partial u_j}\right)_{y_{k=0;k\neq i}}} \#(1)$$

The RGA matrix is:

$$RGA = K_S .* [K_S^{-1}]^T \#(2)$$

$$RGA = [\lambda_{ij}: i, j = 1, \dots, n]$$

. *: is Hadamard multiplication

Among the important proprieties of the RGA matrix is:

the sum of the elements of each column or row is equal to 1.

The choice of the control configuration implies a pair with a gain close to 1 or a positive gain in the case if it exists a negative element in RGA matrix.

fractional method

In this paper, the fractional method used is based on the equivalence between the IMC structure and the conventional feedback structure as shown in figure 1.

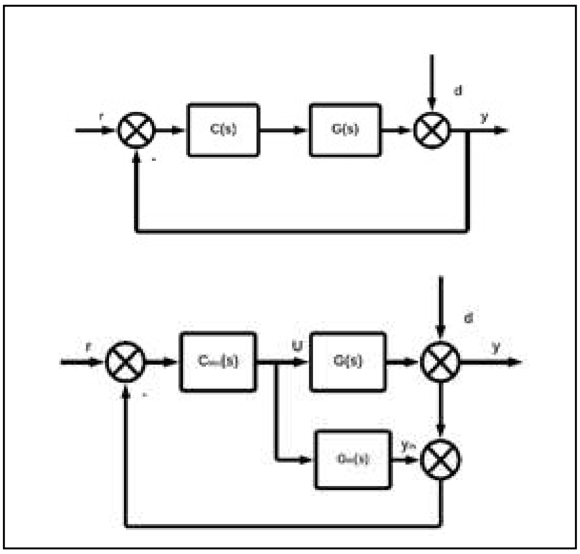

Fig. 1 IMC control scheme and conventional control scheme

Considering a TITO system G(s) which present by:

$$G(s) = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \#(3)$$

There are three steps to design IMC-PID-FO:

• step1: Analyse the interaction using RGA method. We suppose that the model is

$$G_m(s) = \begin{bmatrix} g_{11} & 0 \\ 0 & g_{22} \end{bmatrix} \#(4)$$

• step2: factorize the model in two factors:

$$\mathrm{gm_{ii}}(s) = \mathrm{gm_{ii}^{-}} \mathrm{gm_{ii}^{+}}$$

gm-: the invertible part
gm+: the non-invertible part
• step3: the controller is specified as:

$$C_{imc}(s) = \begin{bmatrix} C_{imc1} & 0 \\ 0 & C_{imc2} \end{bmatrix} \#(5)$$

$$C_{imci}(s) = \frac{f_i(s)}{g_{ii}^{-}}, i = 1,2 \#(6)$$

The reference model is$f_i(s) = \frac{1}{\lambda_i s^{a_i+1}+1}$;$0 < a < 1$ which is the closed-loop Bode's ideal transfer function.
The equivalence between the IMC structure and the conventional feedback structure gives the following result:

$$c(s) = \frac{C_{imci}(s)}{1 - gm(s)C_{imci}(s)} \#(7)$$

The fractional controller obtained with this method has a simple structure. It contains two parts, the first is an integer PID and the second one is a fractional order filter.

3. RESULTS AND DISCUSSION

Distillation column system

The model that will be studied is a distillation column which is took from [16]. The transfer function is presented as:

$$\begin{bmatrix} x_D(s) \\ x_B(s) \end{bmatrix} = \begin{bmatrix} \frac{0.0747e^{-3s}}{12s+1} & \frac{-0.0667e^{-2s}}{15s+1} \\ \frac{0.1173e^{-2.2s}}{11.7s+1} & \frac{-0.1253e^{-2s}}{10.2s+1} \end{bmatrix} \begin{bmatrix} F_R(S) \\ F_v(S) \end{bmatrix} + \begin{bmatrix} \frac{0.70e^{-5s}}{14.4s+1} \\ \frac{1.3e^{-3s}}{12s+1} \end{bmatrix} x_F(s)$$

Structure of controllers

The BLT and the IMC method was applied to design PI controller[17]. The results are illustrated on the following table.

Table 1 PI parameters tuning

Method	Kp,1	Ki,1	Kp,2	Ki,2
BLT	35.789	2.659	-27.207	-3.034
IMC	26.774	2.231	-20.351	-1.995

For the tuning of fractional controller, it is sufficing to follow the steps from equation 3 to 6.
First, Using the RGA method to analyze the interaction:

$$RGA(G(0)) = \begin{bmatrix} 6.0937 & -5.0937 \\ -5.0937 & 6.0937 \end{bmatrix} \quad (8)$$

From the RGA matrix, the best pairing is [u1-y1]; [u2- y2] because the value is positive.
The model is supposed:

$$G_m(s) = \begin{bmatrix} \frac{0.0747e^{-3s}}{12s+1} & 0 \\ 0 & \frac{-0.1253e^{-2s}}{10.2s+1} \end{bmatrix} \quad (9)$$

The parameters of the reference model chosen are: a1=0.2; λ1=4; a2=0.3; λ2=2

The controllers are calculated analytically and the results gotten are:

$$C_1(s) = \frac{12s+1}{0.0747(1+4s^{1.3}-e^{-3s})} \#(10)$$

After the simplification, the final structure of the controller is:

$$C_1(s) = \frac{12}{0.0747}\left(1+\frac{1}{12s}\right).\frac{\frac{1}{4s^{0.2}}}{1+\frac{1-e^{-3s}}{4s^{1.2}}} \#(11)$$

$$C_2(s) = \frac{10.2s+1}{-0.1253(1+2s^{1.2}-e^{-2s})} \#(12)$$

After the simplification of the second controller, the final structure is:

$$C_2(s) = \frac{12}{0.0747}\left(1+\frac{1}{12s}\right).\frac{\frac{1}{4s^{0.2}}}{1+\frac{1-e^{-3s}}{4s^{1.2}}} \#(13)$$

4. SIMULATION RESULTS

set point tracking

Unit step changes were introduced in the first set-point U1=1 and U2=0.the step signal has been started from t=20s. Figure 2 represent the closed loop response of Y1 and figure 3 represent the closed loop response of Y2.

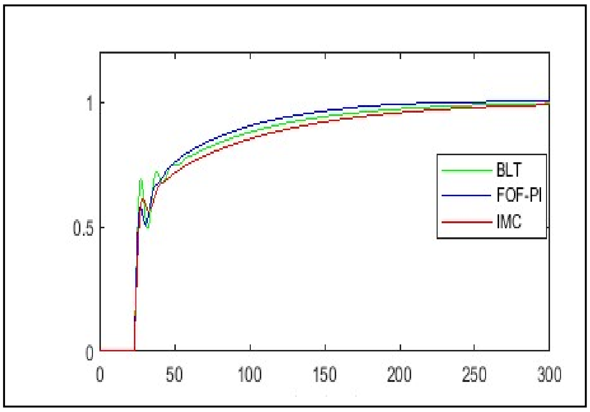

Fig. 2 Step response of y1 for u1=1& u2=0

In figure 2, it has been shown that the response using fractional method is showing better response and has a fast settling time. From figure 3 it is clear that the maximum peak is reached when the BLT method is used and the slowest response is when the IMC method is used.

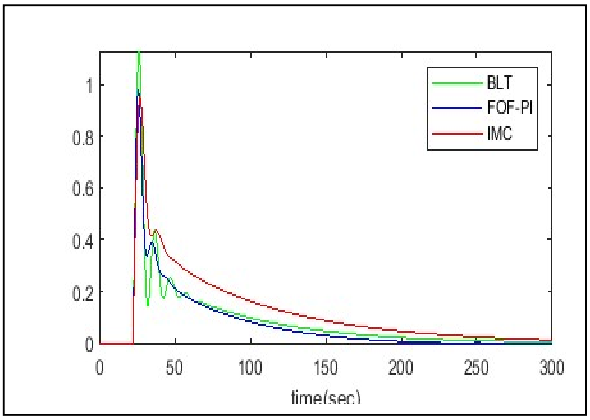

Fig. 3 Step response of y2 for u1=1& u2=0

The table II below presents numerically the performance indices of the step response of y1 and the table 3 shows the performance indices of the step response of y2.
for the first response y1, the settling time and the rise time obtained with fractional method is 211.62 s and 75.7s respectively which is shorter than BLT method and IMC method; the overshoot is 0.11% for the fractional and 0 for other method. It is only little bigger.

Table 2 The performance indices of Y1.

	Y1				
	Max peak	Settling time	IAE	overshoot	Tr(s)
BLT	0.9994	253.348	30.64	0%	90.217
IMC	0.9984	287.1544	36.44	0%	107.90
Fractional	1.0034	211.62	26.52	0.115%	75.70

Table 3 The performance indices of Y2

	Y2			
	Max peak	**Settling time**	**IAE**	**Tr (s)**
BLT	1.1281	231.03	5.15	0.001
IMC	0.9495	298.02	38.14	0.0051
fractional	0.9751	194.73	22.18	0.0122

For the second response and from the table of performance III it is noticed also that the settling time and rise time are shorter for fractional method compared with BLT and IMC method. In addition, the maximum peak reach 1.1281 for BLT method which is greater than the fraction and IMC method.
In the next simulation, the situation is reversed. Unit step changes introduced in the second set-point U2=1 at t=50s and U1=0. Figure 4 presents the closed loop step response of y1 and figure 5 present the closed loop step response for the second loop.

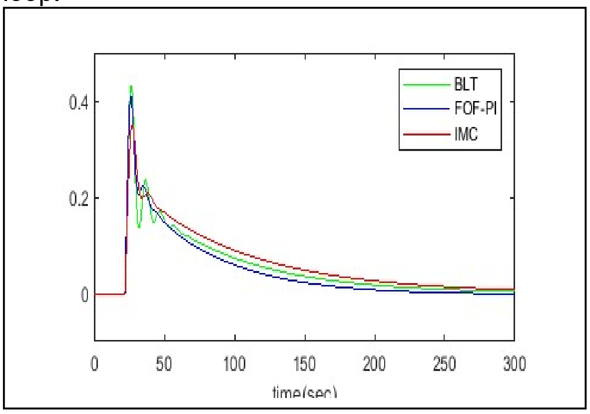

Fig. 4 Step p response of Y1 for u1=0& u2=1

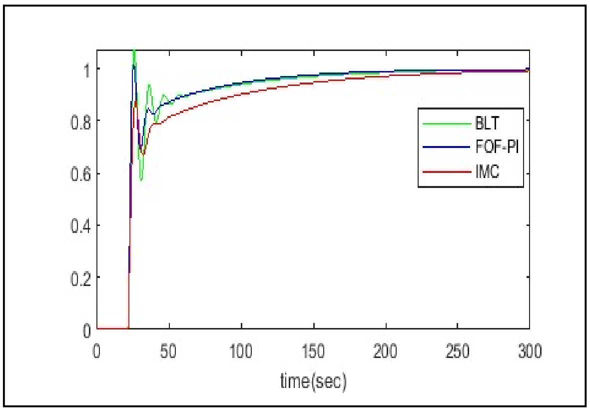

Fig. 5 Step response of Y2 for u1=0 & u2=1

From the step response illustrated in figure 4, it is noticed clearly that the fractional method elaborates fast response and the minimum peak reach when the IMC controller is used but with slowest response. For the second response presented in figure 5, it shown that the response is faster when the fractional and the BLT method is used, But the latter had more oscillations.

The performance indices of y1 and y2 are listed in table IV and table V respectively.

Table. 4 The performance indices of Y1

	Y1			
	Max peak	*Settling time*	*IAE*	*Tr (s)*
BLT	0.4316	275.51	16.31	1.481
IMC	0.3501	326.8	19.39	1.737
fractional	0.4096	235.79	14.08	1.222

Following to the results listed in table 4, it is remarkable that the best performances are linked to the fractional controller specially the settling time and rise time.

Table 5 Performance indices of Y2

	Y2				
	Max peak	*Settling time*	*IAE*	*overshoot*	*Tr(s)*
BLT	1.074	204.65	16.21	8.15%	2.43
IMC	0.999	255.27	24.29	0.43%	72.64
Fractional	1.0157	187.3	14.91	1.531%	2.35

According to the table V, it is noticeable that the overshoot of BLT method is the biggest with 8.15% comparing with fractional and IMC method 1.153% and 0.43% respectively. On the other hand, the settling time and rise time are always shorter with fractional method.

set perturbation rejection

In this part, a unit step signal is introduced in each input. Then at t=300s a step signal is used like a perturbation for a transfer function $\frac{1}{s+1}$ with an amplitude of 0.2. A corresponding closed loop response has been presented in figure 6 and 7 for the responses y1 and y2 respectively.

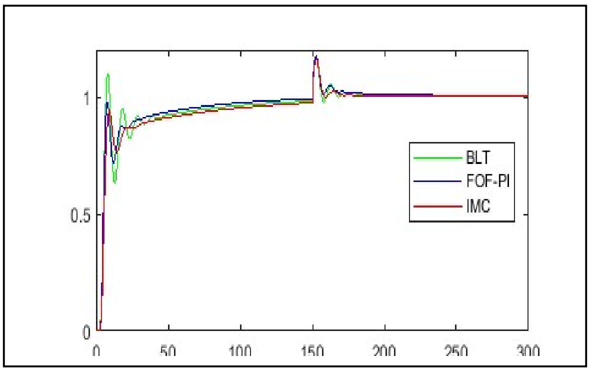

Fig. 6 Step response of Y1 with perturbation at t=150s

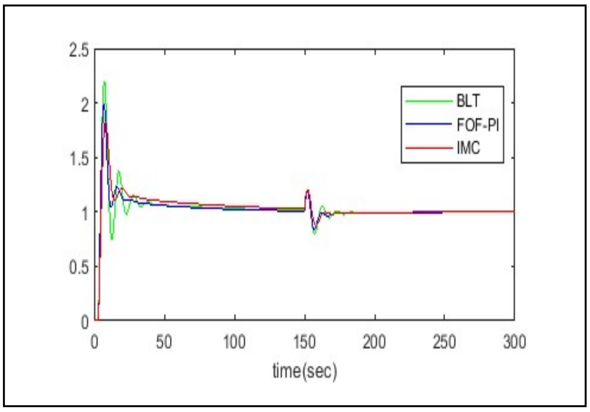

Fig. 7 Step response of Y2 with perturbation at t=150s

From the figure 6 and figure 7, it is cleared that the biggest overshoot appears when we the BLT method is used and for the perturbation, the biggest overshoot is when the IMC method is used. Concerning the response, it is noticed that the response using the IMC method is the slowest. That's what the figure 9 and figure 10 confirmed, which represent the signal effort of the first and the second controllers

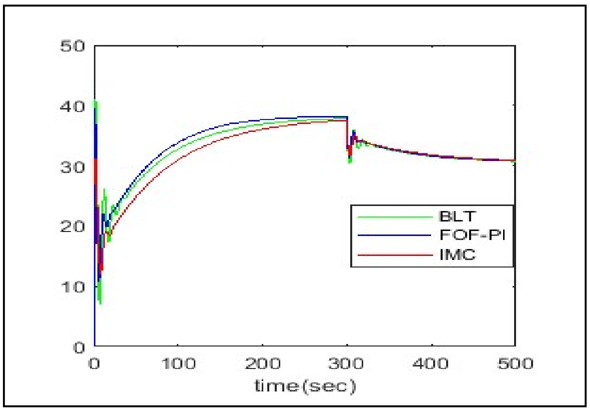

Fig. 8 Signal effort of the first controller

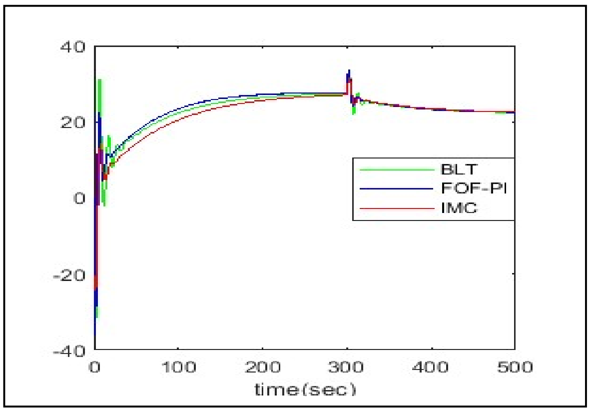

Fig. 9 Signal effort of the second controller

The tables below confirm numerically the performance indices of Y1 and Y2.

Table. 5 Performance indices of y1

Y1					
	Maximum peak	Settling time	IAE	Overshoot	Tr(s)
BLT	1.1798	326.43	17.18	10.35%	2.56
IMC	1.1796	328.073	19.68	0%	3.98
fractional	1.1835	326.99	14.92	0%	2.82

Table. 6 Performance indices of y2

Y2					
	Maximum peak	Settling time	IAE	Overshoot	Tr(s)
BLT	2.2025	329.05	18.81	120.51%	1.15
IMC	1.8243	333.32	13.07	84.25%	1.54
fractional	1.9909	326.21	16.81	99.68%	1.07

To show the effect of the additional fractional order parameter on the step response of the system, the figure 9 and figure 10 present the effect of a2 on y1 and y2.

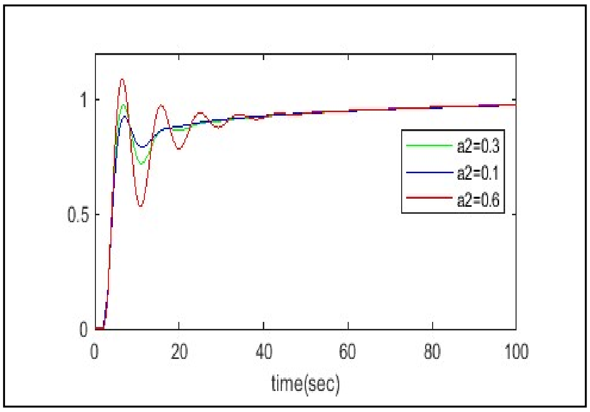

Fig. 10 The effect of a2 on Y1

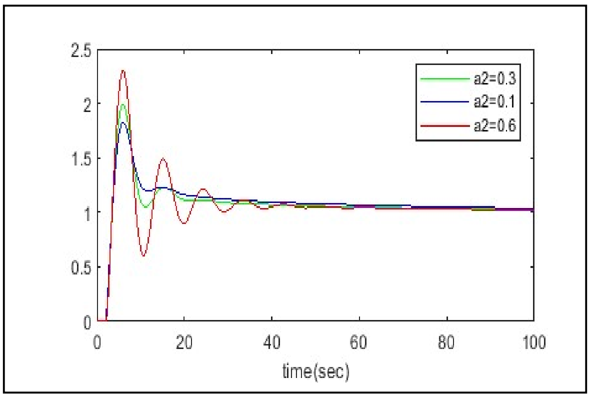

Fig. 11 The effect of a2 on Y2

Concerning the figure 8 and figure 9 illustrate the effect of a2 on y1 and y2. It shows that when the value of a2 increase the oscillations and settling time of both responses y1 and y2 increase too. From the last figures, it noticed that the fractional parameter effect on the oscillation of the response and the settling time.

Finally, the comparative investigation of the closed loop responses of the studied methods conclude that the use of fractional method is better.

5. CONCLUSION

The application of fractional order controller method results an acceptable performance comparing with the integer one where the IMC and the BLT are the methods compared with. This study establishes the superiority and effectiveness of the fractional controller comparing with BLT and IMC ones in the performance results such as the settling time, rise time and integral absolute error. This best performance results of fractional methods implies the best choice of fractional parameters due to its effectiveness on the control quality.

References

[1] J. Ziegler and N. Nichols. “Optimum setting for automatic controllers”. Trans.ASME, vol. 64, pp. 759-768, 1942.

[2] Y. Zhang, Q. Wang and K. Astrom. “Dominant pole placement for multi-loop control systems”. Automatica, vol. 38, pp. 1213-1220, 2002

[3] W. Luyben. “Simple method for tuning SISO controllers in multivariable systems”. Industrial&Engeneering Chemistry Process Design and Developpement, vol. 25, pp. 654-660, 1986.

[4] W. Luyben, “Process modeling simulation and control for chemical engineers”, McGraw-Hill Higher Education, 1989.

[5] C. Economou and M. Morari. “Internal model control: multiloop design”. Industriel&Engineering Chemistry Process Design and Development, vol. 25, pp. 411-419, 1986.

[6] Y.zennir, e.guechi, L.chetioui, R.bendib. “ IMC multi-controller strategies applied on manipulator robot”, Algerian journal of signals and systems, vol.2, pp.51-62, 2017.

[7] I. Podlubny. “Fractional-order systems and piλdµ controller”. IEEE Transactions on automatic control, pp. 208-214, 1999.

[8] M. Bettayeb and R. Mansouri. “IMC-PID-fractional-order-filter controller design for integer order systems”. ISA Transactions, vol. 53, pp. 1620-1628, 2014.

[9] C.Suryaprakash, J.M.Kishorelal, U.Gururagavan, S.Gokulan and AJITH.B.SINGH. “FOPID implementation for industrial process”. international journal of innovative research in electrical, electronics, instrumentation and control engineering, vol. 2, pp. 1420-1424, 2014.

[10] P. Mishra, V. Kumar and K. Rana. “A fractional order fuzzy PID controller for binary distillation column control”. Expert Systems with Applications, vol. 42, pp. 8533-8549, 2015.

[11] G.Prakash and V.Alamelumangai. “Design of Predictive Fractional Order PI Controller for the Quadruple Tank process”. WSEAS TRANSACTIONS on SYSTEMS and CONTROL, vol. 10, pp. 85-94, 2015

[12] A. Numsomran, T. Trisuwannawat and V. Tipsuwanporn. “Adaptive Fractional Order PIλ Dµ Control for Multi-Configuration Tank Process (MCTP) Toolbox”. International journal of intelligent engineering & systems, vol. 11, pp. 21-34, 2018.

[13] y.Zennir, e.Mechhoud, A.Seboui, R.bendib. “optimal PSO-PIDmulti-controller for a robotic wrist”. Algerian Journal of signals and systems, vol. 3, pp.22-34,2018

[14] T. Chekari, R. Mansouri and M. Bettayeb. “IMC-PID Fractional Order Filter Multi-Loop Controller Design for Multivariable Systems Based on Two Degrees of Freedom Control Scheme”. International journal of control, automation and systems, vol. 16, pp. 689-701, 2018.

[15] E. Bristol. “On a new measure of interaction for multivariable process control”. IEEE transactions on automatic control, vol. 11, pp. 133-134, 1966.

[16] D. Seborg, T. Edgar and D. Mellichamp, “process dynamics and control”. United States of America: John& Sons,Inc, 2004.

[17] R. Totuk, A. S. Nurul, S. Nurulspika and Z. Haslinda. “PID multivariable tuning system using BLT method for distillation column”. Materials Science and engineering vol. 458, 2018.