IMC-PID-FOF Multi-loop controller design for Binary Distillation Column

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Abstract: This paper deals with the fractional order multi-loop controller design for a distillation column. The idea is a generalization of the IMC-PID-FOF controller design method developed for monovariable systems to the distillation column model which is multivariable. The principle is based on the IMC paradigm and the choice of appropriate control configuration with minimum of interactions. The proposed method is illustrated with an example of a distillation column model taken from the literature.

Keywords: distillation column model, IMC control, Bode's ideal transfer function, fractional order control, multi-loop controller.

1. INTRODUCTION

The distillation is undoubtedly the most important separation method in the chemical process industries with 95% of the separation work is carried out by distillation and these units consume 3% of the total energy produced in the world [1]. From the point of view analysis and control, the distillation column is considered as a strongly nonlinear and very interactive multivariable system and generally with very high order system [2]. Therefore, the distillation column is still an attractive research subject for the researchers and control engineers for testing and validating their proposed controller design methods [3-5]. The elaboration of linear dynamic model for the distillation column is an important part in the control system synthesis; it can be obtained using the system identification which allows avoiding a complicated and expensive nonlinear model. However the time constants of the compositions dynamics are large and a recording of input / output data for the real plant is very time consuming. Moreover, each experiment causes undesired disturbances of the product qualities and it is practically impossible to obtain models for the entire operating range of the distillation column [6]. These disadvantages lead to the recommendation of the second method that means the linearization of the non linear model [7].

During the last decades, the Fractional Order Control (FOC) knows a growing interest in the control system theory field, the early research works are carried out by Podlubny [8] where he introduced the notion of FO-PID controller and Oustaloup [9] where he introduced the notion of CRONE control (the French acronym which means non-integer order robust control). In [10], an IMC-PID-FOF controller is proposed for monovariable systems. The principle is based on the Internal Model Control (IMC) paradigm which exhibits interesting properties such as robustness to modeling error. Moreover, the controller design method is very simple with few parameters to tune [11]. Other research works are dealing with fractional order controllers design for multivariable systems such as [12-16].

This paper presents a generalization of IMC-PID-FOF controller design method developed for monovariable systems [10] to distillation column models which are multivariable systems. This proposed method is also a generalization of the IMC-PID multi-loop controller design method introduced by Economou and Morari [17] to the case when the reference model is of fractional order; it is based on the equivalence between the IMC and conventional multi-loop structures. The design method consists of the stability analysis of the closed loop multi-loop structure [18] and the interactions analysis to select the appropriate pairing with a weak level of interactions [19]. Then, an independent IMC-PID-FOF controller is determined for each control loop. The fractional property of the controller is imposed by the closed loop Bode's ideal transfer function chosen as reference model for each loop. The rest of the paper is organized as follows: Section (2) deals with some definitions relative to the distillation column in terms of modeling and control. The proposed design method is presented in Section (3) which is illustrated with an example of distillation column model in Section (4). A conclusion of the study is given in Section (5).

2. DESCRIPTION AND MODELING OF THE DISTILLATION COLUMN

A typical two-product distillation column is shown in Fig. 1. It contains a vertical column where trays are used to enhance the component separations, a re-boiler to provide heat for the necessary vaporization from the bottom of the column, a condenser to cool and condense the vapor from the top of the column, and a reflux drum to hold the condensed vapor so that liquid reflux can be recycled back from the top of the column. The feed product to be split is given by F, the feed rate [kmol/min], and Z_f its composition [mole fraction]. The resulting products to be extracted from the top and bottom of the column are given respectively by D and B which are respectively the distillate and the bottom products flow rate [kmol/min]. x_D and x_B are respectively distillate and bottom products composition which refers usually to the amount of light component [mole fraction] in distillate and bottom product respectively. *L* and *V* represent respectively the reflux and voilup flows [kmol/min]. M_B and M_D represent respectively the liquid holdup in the re-boiler and the condenser.

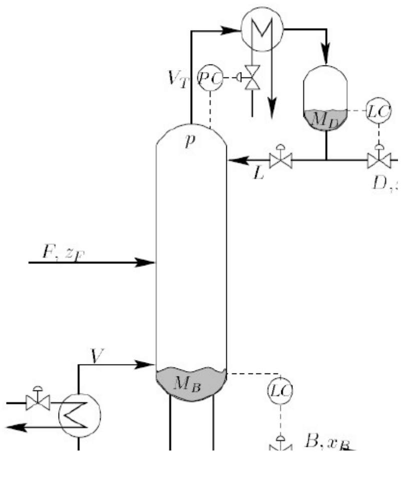


Fig. 1 Binary distillation column controlled with LV-configuration.

The principle of the separation is based on the differences in the boiling points of the components in the feed product, in such a way that the more volatile component rises at the top of the column and the less volatile component goes down at the bottom [2].
The relative volatility is a measure of the differences in volatility between two components and hence their boiling points, it indicates how easy or difficult a particular separation will be, the relative volatility of the component i with respect to component j is defined as [6]

$$\alpha_{ij} = \frac{\left[\frac{y_i}{x_i}\right]}{\left[\frac{y_j}{x_j}\right]} = \frac{k_i}{k_j} \qquad (1)$$

y_i is the mole fraction of component i in the vapor phase and x_i is the mole fraction of component i in the liquid phase.
The mathematical model of the distillation column involves the equations of the energy and material balances in each tray, the model of the liquid flow dynamics (changes in the liquid holdups) and the model of the pressure dynamics. It may also include detailed model of the reboiler and the condenser. The resulting model is thus called rigorous and highly nonlinear and generally with very high order. However, in this rigorous model a number of simplifications are included such the perfect mixing in both phases on all stages, thermal and thermodynamic equilibrium between the phases.
To simplify moreover this model a number of assumptions are considered [6]:

1- The relative volatility α is constant throughout the column; this means that vapor equilibrium relationship can be expressed as

$$y_n = \frac{\alpha\, x_n}{1+(\alpha-1)x_n} \qquad (2)$$

x_n and y_n are respectively liquid and vapor composition on the n^{th} tray.

2- The overhead vapor is totally condensed in the condenser
3- The liquid holdups on each tray, condenser and the re-boiler are constant
4- The vapor holdup is negligible throughout the system
5- The molar flow rates of the vapor and liquid through the stripping and rectifying sections are constant

Due to the Vapor Liquid Equilibrium (VLE) relationship given in Equation (2), the distillation column model is still non linear and may be of high order. Thus the methods of linearization and reduction model can be used [2].

Two-product distillation column has five degrees of freedom with five manipulated variables U = (*L*, *V*, V_T, *D*, *B*) and five controlled outputs Y = (x_D , x_B , M_D, M_B, *P*). Many studies have shown that the process has poles in or close to the origin and needs to be stabilized and for high purity distillation, the system is strongly interactive. For this, the distillation column is first stabilized by closing three decentralized loops for levels and pressure involving the outputs: Y_2 = (M_D, M_B, *P*). These three SISO loops are usually interact weakly and may be tuned independently of each other; the remaining outputs are then the product composition: Y_1 = (x_D, x_B).
There exist many possible choices for U_2 to control Y_2 and thus for U_1 to control Y_1. Par convention, each configuration is named by the inputs U_1left for the composition control such as:

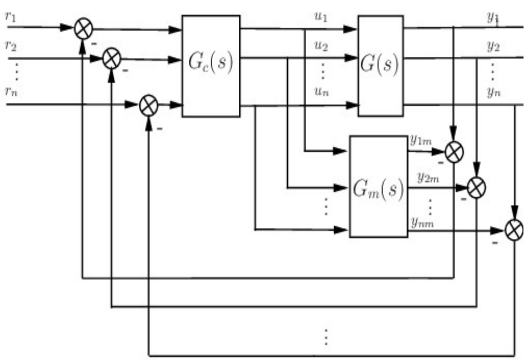
- LV configuration: U_1 = $(L\ V)^T$ and U_2 = $(D\ B\ V_T)^T$
- DV configuration: U_1 = $(D\ V)^T$ and U_2 = $(L\ B\ V_T)^T$

3. IMC-PID-FOF MULTI-LOOP CONTROLLER FOR DISTILLATION COLUMN MODEL

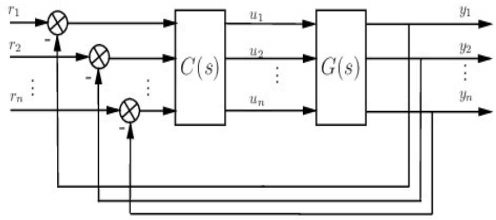
The IMC multi-loop control scheme [17] is generalized to the fractional order case whose fractional order model is chosen.
In the frequency domain, most distillation column processes are modeled by transfer function matrices.
We assume that a distillation column is represented by a transfer function matrix with *n* inputs and *n* outputs

$$G(s) = \begin{pmatrix} g_{11} & \cdots & g_{1n} \\ \vdots & \ddots & \vdots \\ g_{n1} & \cdots & g_{nn} \end{pmatrix} \qquad (3)$$

The IMC and conventional multi-loop control schemes are shown in Fig. 2



(a)



(b)

Fig.2 IMC and conventional multi-loop structures

The IMC-PID-FOF multi-loop controller consists of:

- Step1 : Stability and interactions analysis

To verify the stability of the closed loop response of the distillation column when a muti-loop control scheme is implemented, the Niederlinski Index NI is used [18].

$$\mathrm{NI} = \frac{\det[G(0)]}{\prod_{i=1}^{n} g_{ii}(0)} \qquad (4)$$

$$G(0) = \lim_{s \to 0} g_{ii}(s)$$

If NI > 0; the controlled distillation process will be stable.

If NI < 0; the controlled distillation process will be unstable.

The Relative Gain Array (RGA) measure is used to select the adequate pairing with least of interactions [19], it is given by

$$RGA\big(G(0)\big) = G(0) \otimes \left(G(0)^{-1}\right)^T \qquad (5)$$

Where ⊗ denotes the Hadamard product and the superscript *T* designates the transpose of a matrix.
For the sake of simplicity, a control configuration that assigns each input u_i to output $y_i$is assumed for i=1…. n, the process model is

$$G_m(s) = \begin{pmatrix} g_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & g_{nn} \end{pmatrix} \qquad (6)$$

In Equation (6), the interaction elements are dropped and are considered as modelling errors [16]. The multivariable process is thus considered as a set of monovariable subsystems (loops) and monovariable controller is designed for each loop independently from others.

- Step 2 : IMC multi-loop controller synthesis

The IMC multiloop controller is given by

$G_c(s) = diag[g_{c1}(s), g_{c2}(s) \dots\dots g_{cn}(s)]$ (7)
In order to calculate each controller $g_{ci}(s)$, i=1,...,n, the corresponding element in the process model is first factorized as

$g_{ii}(s) = g_{ii}^{-}(s) g_{ii}^{+}(s)$ (8)

The controller is then given by

$g_{ci}(s) = \frac{1}{g_{ii}^{-}(s)} f_i(s)$, i=1,.......n. (9)

A fractional behavior is imposed for each loop and $f_i(s)$ is given by the closed loop bode's ideal transfer function

$f_i(s) = \frac{1}{1+\tau_{ci} s^{\alpha_i+1}}$ 0<α_i<1; i=1,...n (10)

This function exhibits interesting properties: the infinite gain margin and the constant phase margin: $\varphi_m = \pi(1 - \frac{\alpha_i}{2})$ dependent only on α_i. The time constant τ_{ci} determines the settling time of the step response and the order α_i determines the overshoot. Therefore, the system is robust to process gain variations and step response exhibits iso-damping property.

- Step 3 : conventional multi-loop controller design

The classical multi-loop controller to be implemented is

$C(s) = diag[c_1(s), c_2(s), \dots\dots, c_n(s)]$ (11)

The equivalence between the two structures of Fig. 2 gives each element of $c_i(s)$ as

$c_i(s) = \frac{g_{ci}(s)}{1-g_{ci}(s)\, g_{ii}(s)}$, i=1.....n (12)

4. Simulation results

To illustrate the controller design method for distillation column models, We consider a cryogenic Carbon isotope separation column model. The Carbon isotopes separation process is based on the distillation of Carbon monoxide which has different boiling temperatures depending on the Carbon isotopes it contains.

A three input-three output system is identified where the inputs variables are: the output waste flow from the column, the input feed flow to the column and the electrical power supplied to the boiler resistor. The outputs variables are: the pressure in the column at the condenser zone, the liquid Carbon monoxide level in the boiler and the pressure at the boiler zone [13, 20].

$$G(s) = \begin{pmatrix} g_{11}e^{-10s} & g_{12}e^{-10s} & 0 \\ g_{21}e^{-10s} & g_{22}e^{-8s} & g_{23} \\ g_{31}e^{-18s} & g_{32}e^{-35s} & g_{33} \end{pmatrix} \quad (13)$$

With:

$$g_{11} = \frac{-0.1111}{s^2 + 1.0945s + 0.08423}$$

$$g_{12} = \frac{0.1152}{s^2 + 1.211s + 0.2021}$$

$$g_{21} = \frac{-0.001731}{s^2 + 0.1343s + 0.001961}$$

$$g_{22} = \frac{0.003846}{s^2 + 0.1547s + 0.004357}$$

$$g_{23} = \frac{-1.104}{s+0.1176}\ ; \quad g_{33} = \frac{8.457}{s+0.9851}$$

$$g_{31} = \frac{-0.009918}{s^2 + 1.056s + 0.07036}$$

$$g_{32} = \frac{0.006288}{s^2 + 1.085s + 0.09851}$$

According to Equation (4), the corresponding Niederlinski Index is: NI=0.5715 and using Equation (5), the corresponding RGA matrix is:

$$RGA = \begin{pmatrix} 1.8883 - 0.8883 & & 0 \\ -0.7562 & 1.7499 & 0.0063 \\ -0.1321 & 0.1384 & 0.9937 \end{pmatrix} (14)$$

This positive value of NI shows that the distillation column will be stable when multi-loop control scheme is implemented and the RGA matrix given by Equation (14) shows that the diagonal pairing is appropriate to control the distillation column: y_i is paired with u_i ; i=1,2,3. Consequently, the process model without interactions that will be used in the controller design method is

$$G_m(s) = \begin{pmatrix} g_{11}e^{-10s} & 0 & 0 \\ 0 & g_{22}e^{-8s} & 0 \\ 0 & 0 & g_{33} \end{pmatrix} \quad (15)$$

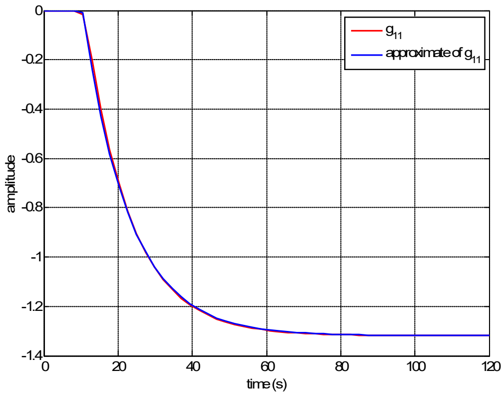
In Equation (15), the off-diagonal elements are dropped and considered as modelling errors.

The transfer functions $g_{11}$and $g_{22}$are of second order; in order to simplify the fractional order multi-loop controller design method, $g_{11}$and $g_{22}$are identified as first order models

$$g_{11}e^{-10s} \approx \tilde{g}_{11} = \frac{-1.319e^{-10.5s}}{1+12.4s} \quad (16)$$

$$g_{22}e^{-8s} \approx \tilde{g}_{22} = \frac{0.8827e^{-13.3s}}{1+30s} \quad (17)$$

The step responses of g_{11}; g_{22} and their approximates $\tilde{g}_{11}$; $\tilde{g}_{22}$ respectively are given in Fig.3.



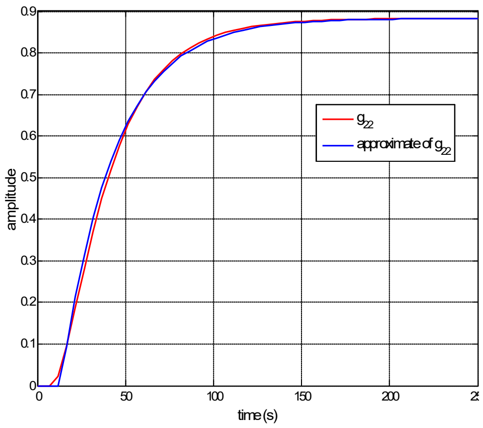


Fig.3 Step responses of g_{11} and $\tilde{g}_{11}$, g_{22} and $\tilde{g}_{22}$

We see, from Fig.3, that the temporal behaviour of g_{11}; $\tilde{g}_{11}$ and g_{22}; $\tilde{g}_{22}$ respectively is very close. This confirmed by the small maximum error value between g_{11} and $\tilde{g}_{11}$: 0.0356 and that between g_{22} and $\tilde{g}_{22}$: 0.0339.

The IMC-PID-FOF multi-loop controller is designed according to the steps described in Section (3) using the following process model

$$G_m(s) = \begin{pmatrix} \tilde{g}_{11} & 0 & 0 \\ 0 & \tilde{g}_{22} & 0 \\ 0 & 0 & g_{33} \end{pmatrix} \quad (18)$$

The performance of the proposed IMC-PID-FO multi-loop controller is compared to the fractional order multi-loop controller proposed in [13]. The parameters values of the reference model $f_i(s)$, i=1,2,3 are listed in Table 1.

Table 1 Parameters of the reference model

Loop i	$f_i(s)$	
	τ_{ci}	α_i
1	135.6457	0.2222
2	81.682	0.2556
3	0.4749	0.2667

These values are chosen to meet the same specifications for each loop as given in [13]

According to Equations (7) to (12), the numerical expression of the IMC-PID-FOF multi-loop controller is

$$C(s) = \begin{pmatrix} C_1(s) & 0 & 0 \\ 0 & C_2(s) & 0 \\ 0 & 0 & C_3(s) \end{pmatrix} \quad (19)$$

With:

$$C_1(s) = \frac{-0.8953}{1 + 12.9186\, s^{0.2222}}\left(1 + \frac{1}{12.9186s}\right)$$

$$C_2(s) = \frac{2.5554}{1 + 6.1415\, s^{0.2556}}\left(1 + \frac{1}{30s}\right)$$

$$C_3(s) = \frac{0.249}{s^{0.2667}}\left(1 + \frac{1}{1.0151s}\right)$$

The fractional order integrals and derivatives terms are implemented in Matlab using the Oustaloup's continuous approximation method with frequency range $[10^{-5}, 10^{+2}]$ using 15 cells.

To illustrate the effect of interactions among the control loops, sequential set-point changes are made in the first and second loops when proposed and auto-tuning controllers are implemented and the obtained results are given in Fig. 4 and Fig. 5.

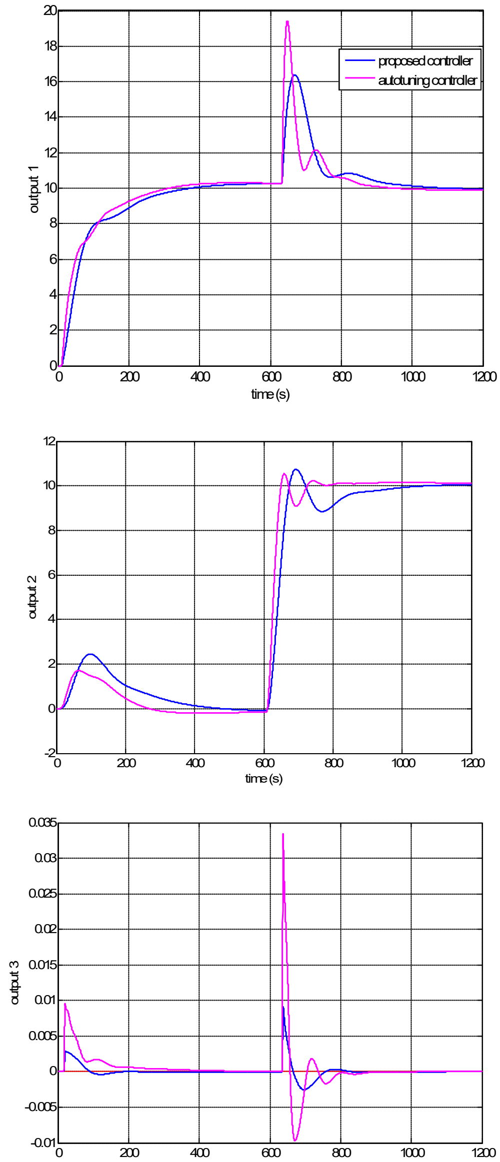


Fig.4 Closed loop responses to sequential changes in the first and second set-points

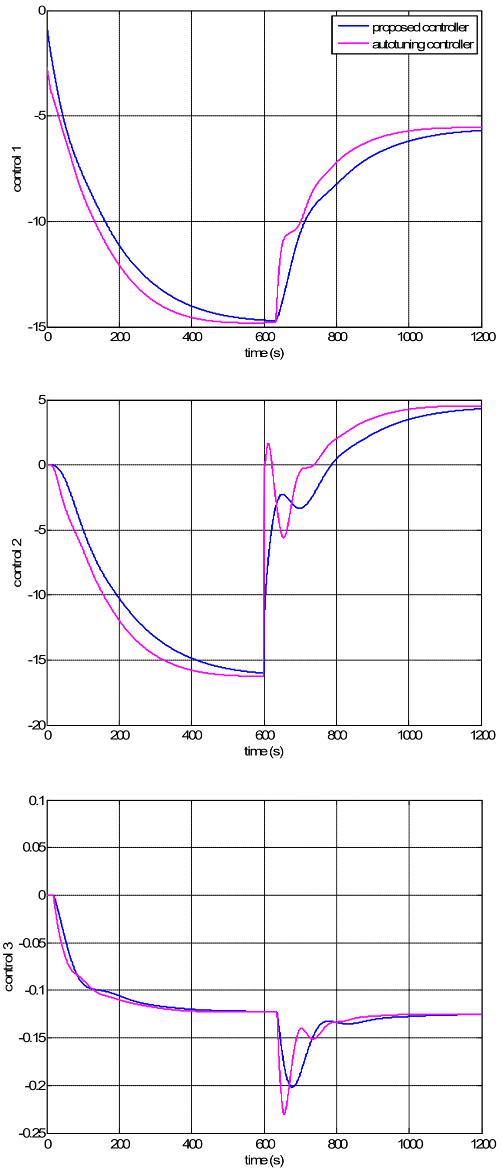


Fig. 5 Control effort to sequential changes in the first and second set-points

The simulations results of Fig. 4 show that both controllers ensure set-point tracking with close behaviour of the two responses with a superiority of the proposed controller concerning the reduction of effect of interactions in the first and third loops. However, the interactions are more reduced for second loop when auto-tuning controller is implemented. On the other hand, the value of Integral Absolute Error (IAE), obtained for each loop, show the superiority of the auto-tuning controller for the first and second loop as given in Table 2. The IAE value for the third loop is smaller when proposed controller is implemented.

Table 2 Integral Absolute Error (IAE) for each loop

	proposed controller	Auto-tuning controller
IAE1	1450	1264
IAE2	996.1	664
IAE3	0.4448	1.342

The obtained results of Fig. 5 show that the control effort provided by the two controllers is acceptable.

Simulation results of Fig.6 and Fig. 7 show the closed loop responses with $\pm 50\%$ of all gains when proposed and auto-tuning controllers are implemented respectively.

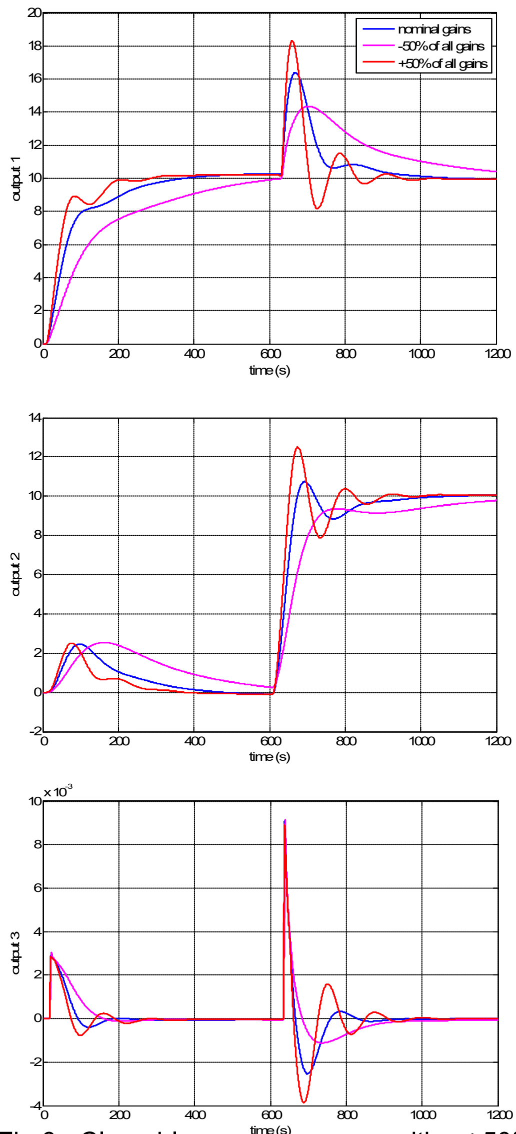


Fig.6 Closed-loop responses with $\pm 50\%$ variation on all process model gains when IMC-PID fractional multi-loop controller is implemented

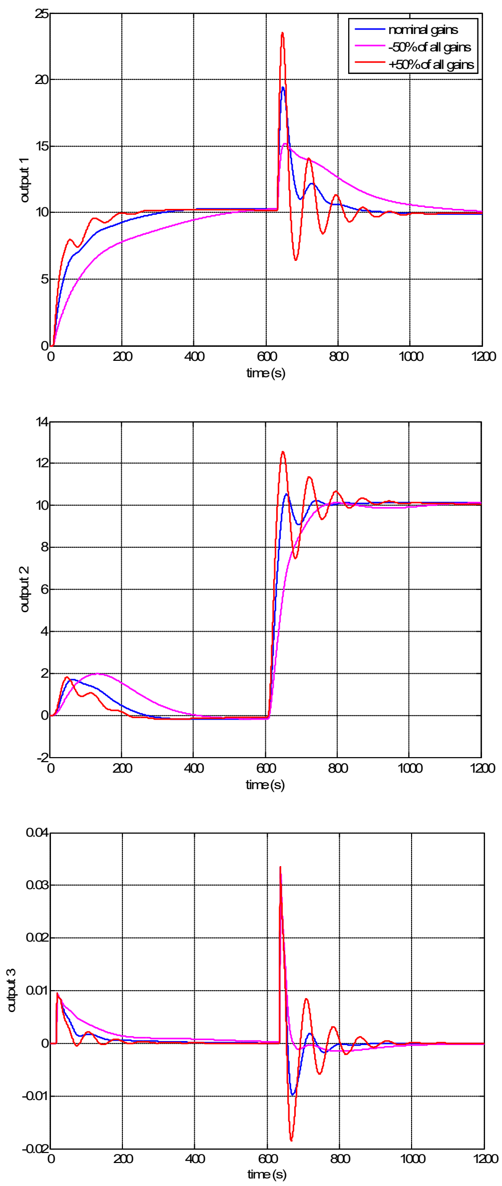


Fig.7: Closed-loop responses with $\pm 50\%$ variation on all process model gains when auto-tuning fractional multi-loop controller is implemented

We see from the simulation results of Fig.6 and Fig.7 that both controllers are robust to process gain variations.

5. CONCLUSION

In this paper, an IMC-PID-FOF multi-loop controller is designed for distillation column model which is a multivariable and highly interactive model. The controller design method is very simple with few parameters to tune fractional order α_i and time constant τ_{ci}

The simulation results obtained, when proposed controller is compared to another fractional order controller, show that the IMC-PID-FOF multi-loop controller ensures set-point tracking and exhibits robustness to process gain variations. Future works will deal with reducing the effect of interactions among the control loops in distillation column and optimize the choice of the parameters α_i and τ_{ci} ; the proposed controller will designed to control the top and bottom products purity in the distillation columns.

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