Fractional order robust adaptive control design for actuator failure compensation in dual ball-beam system

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Abstract: This paper is to propose a fractional order adaptive control scheme design for actuator failure compensation in order to balance of a ball-beam system with redundant actuators. The proposed controller is based on a fractional order sliding surface from which an adaptive actuator failure compensation control scheme is obtained. In order to control the dual ball-beam system with redundant motors, a linearized model is derived, which is subjected to actuator failures. A set of numerical simulation tests; representing few selected types of actuator failures scenarios are carried out to assess the output tracking performance of the proposed controller design when dealing with such failures.

Keywords: Fractional order control, Actuator failures, Adaptive sliding mode, redundant actuators, ball-beam system.

1. INTRODUCTION

With the constant rapid development of technology, an increasing demand in performance and reliability of flight control systems was observed, however despite these advancements one can still encounter potential problems or failures on systems or components such as sensors or actuators that can severely reduce the controlled systems performances [1]. Actuator failure problem is substantially more critical in aircraft and flight control systems as it may lead to catastrophic human and material losses [2].

For that matter, a growing interest has been granted to the problem of designing control strategies for the accommodation of failures and especially for actuator failures compensation, and consequently great progress has been made in this field, a popular approach for the design of fault tolerant control and actuator failure compensation is adaptive control [3,4] due to the fact that adaptive controllers are capable to handle such failures by controller parameters readjustment, as a matter of fact several effective control strategies have been proposed to deal with this issue: in [5] authors proposed an Indirect Adaptive Fault-Tolerant approach for Attitude Tracking of Spacecraft that is robust to external disturbances and modelling uncertainties. In [6] and [7] an adaptive control scheme for nonlinear systems with unknown actuator failures was developed using feedback linearization techniques, furthermore the authors in [8] an adaptive fuzzy controller is proposed using back-stepping technique with uncertain actuator faults, concerning aircraft fault tolerant control other methods have also been successfully implemented such as adaptive sliding mode control for flexible spacecraft [9], robust adaptive state feedback controller for rocket fairing structural-acoustic model [10].

In recent years, more focus was directed towards the use of fractional order control in failure accommodation: in [11] the authors proposed a fault tolerant adaptive sliding mode synchronization scheme for a class of fractional-order non-linear systems, and in [12] fractional-order adaptation laws were introduced, for an adaptive controller design based on a modified back-stepping technique. Moreover, the authors in [13] effectively incorporated neural networks in the development of a fractional-order back-stepping adaptive fault-tolerant control method for the attitude synchronization tracking of multi-UAV. Despite these recent breakthroughs and the interesting properties of fractional order controllers in the improvement of system performances in term of behavior, response time and disturbance rejection [14,15] there is still significant research to be made in regards to the study of fractional order fault tolerant control.

In the following, our task will be to establish an effective scheme for actuator failure compensation based on fractional calculus. The remainder of the paper will be organized as follows: We will first present the linearized model of a ball-beam system with redundant actuators along with the modelling of the actuator failures to be accommodated, then we will define a fractional order sliding surface from which we derive an adaptive fault tolerant control scheme, finally we will carry a number of simulations to evaluate the efficiency of this proposed control strategy against actuator faults, a conclusion regarding the obtained results and future works will be presented.

2. PROBLEM STATEMENT

System description

The dual-actuator ball-beam system consists of two motors on each of the beam driving its inclination, which are redundant, meaning they can perform the same function, the objective of the two motors is to balance a ball at a desired position on the beam by moving the two ends up and down [16],a description of the system is shown in Fig. 1

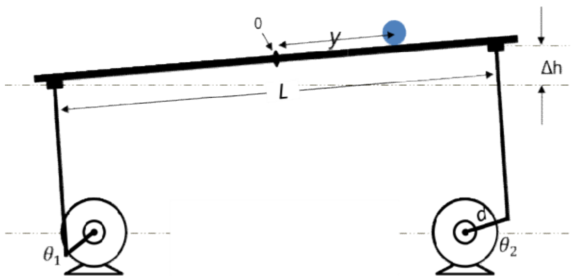

Fig. 1 Representation of a dual motor ball-beam system

Under the assumption that the beam offset Δh is negligible compared to the beam length, a linearized state space representation with simplified actuator dynamics of the ball beam system is

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1)$$

With the state matrices defined as

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ \frac{-mgdr^2 a_1}{L(J+mr^2)} & \frac{mgdr^2 a_2}{L(J+mr^2)} \end{bmatrix} \quad (2)$$

Where **m** and **r** represent the mass and the radius of the ball respectively, **g** is the gravitational force, **d** is the lever offset, **L** is the total length of the beam, **J** is the moment of inertia of the ball, and $\mathbf{a_i}$ represents the proportional gain of the $\mathbf{i^{th}}$actuator. and the input is the voltage signal applied to both motors with which have opposite effect on the ball position since we have $\mathbf{b_{2,1} = -b_{2,2}}$in the input matrix B [17], and the output of the system is taken as the position of the ball

$$y(t) = x_1(t) \quad (3)$$

Actuator failure modeling

In the literature, several representations of actuator failures are considered such as parametrizable time-varying failures and unparameterizable time-varying failures [3], let us consider the following actuator failure model

$$u_j(t) = u^f{}_j(t) \qquad t \geq t_j \quad (4)$$

The expression (4) signifies that at time instant $t = t_j$ the j^{th} actuator fails, it is worth noting that the value of $u^f{}_j(t)$, its pattern and time of occurrence are unknown, we can express the input as the following

$$u(t) = v(t) + \sigma\left(u^f(t) - v(t)\right) \quad (5)$$

Where $v(t)$ denotes the controller signal, σ is a diagonal matrix representing fault indexes where$\sigma_j = 1$ if the j^{th} actuator fails and $\sigma_j = 0$, by rearranging (5) we get

$$u(t) = (1-\sigma)v(t) + \sigma u^f(t) \quad (6)$$

The second state can be written as

$$\begin{aligned} \dot{x}_2(t) &= b_1 u_1(t) + b_2 u_2(t) \\ &= \sum_{j=1}^{m} b_j . u_j(t) \end{aligned} \quad (7)$$

With m=2 the number of redundant actuators. Replacing (6) into (7) gives us the following expression

$$\dot{x}_2(t) = \overline{b_h} v(t) + \overline{b_f}\,(u^f) \quad (8)$$

With $\overline{b_h} = \sum_{j=1}^{m} b_j \left(1 - \sigma_j\right)$ and $\overline{b_f}\,(u^f) = \sum_{j=1}^{m} b_j \sigma_j u_j^f(t)$.

3. FAULT TOLERANT CONTROLLER DESIGN

3.1 Fractional calculus preliminaries

Fractional calculus is the generalization of integration and differentiation to non-integer order [18], fractional order operators are commonly represented by ${}_a\mathcal{D}_t^{\alpha}$ where α denotes the non-integer order, a and t are the limits of the operator, it can be used to represent both fractional derivative and fractional integral based on the value of α:

$$ {}_a\mathcal{D}_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha} \alpha > 0 \\ 1 \qquad \alpha = 0 \\ \int_a^t (dt)^{-\alpha} \alpha < 0 \end{cases} $$

Several definitions of non-integer differentiation and integration are used, one popular definition is the so-called *Riemann-Liouville* definition which is given as [19]

$$ {}_a\mathcal{D}_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{\alpha-1} f(\tau) d\tau \tag{9} $$

for the fractional order integral and as

$$ {}_a\mathcal{D}_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t (-\tau)^{n-\alpha-1} f(\tau) d\tau \tag{10} $$

or the fractional order derivative, with $n-1 < \alpha < n$ and $\Gamma(.)$ denotes the Euler Gamma function defined by [20]

$$ \Gamma(\mathrm{x}) = \int_0^\infty e^{-t} t^{x-1} dt \tag{11} $$

3.2 Fractional order sliding control law

Our objective is to develop a fractional control law that can guarantee that the output of the system will be able to track a given reference with acceptable performances even in the event of unexpected actuator failures, for that purpose let us define the output tracking error

$$ e(t) = y_r(t) - y(t) \tag{12} $$

We propose the following fractional order sliding surface

$$ s(t) = \mathcal{D}^\alpha e(t) + \lambda e(t) \tag{13} $$

Assuming zero initial conditions $e(0) = 0$ and using fractional derivative properties, we can write the error as [21] $e(t) = \mathcal{D}^{-1}\dot{e}(t)$ thus we get

$$ s(t) = \mathcal{D}^{\alpha-1}\dot{e}(t) + \lambda e(t) \tag{14} $$

By taking the time derivative of (13) we obtain

$$ \dot{s}(t) = \mathcal{D}^{\alpha-1}\ddot{e}(t) + \lambda\dot{e}(t) \tag{15} $$

We replace the error by its expression

$$ \dot{s}(t) = \mathcal{D}^{\alpha-1}\left(\ddot{y}_r(t) - \left(\overline{b_h} v(t) + \overline{b_f}\,(u^f)\right)\right) + \lambda\dot{e}(t) \tag{16} $$

If we arrange equation (15) and by grouping all additional terms it becomes

$$ \dot{s}(t) = -\mathcal{D}^{\alpha-1}\left(\overline{b_h} v(t) + \overline{b_f}\,(u^f)\right) + f(t) \tag{17} $$

With $f(t) = \mathcal{D}^{\alpha-1}\ddot{y}_r(t) + \lambda\dot{e}(t)$. Hence, we obtain the following fractional control law

$$ v_{eq}(t) = \overline{b_h}^{-1}\left(\overline{b_f}\,(u^f) + \mathcal{D}^{1-\alpha} f(t)\right) + \overline{b_h}^{-1}\left(k_1 \mathcal{D}^{1-\alpha} s(t) + k_0 \mathcal{D}^{1-\alpha} sign(s)\right) \tag{18} $$

By replacing the equivalent control input (17) in the expression (16) we finally get

$$ \dot{s}(t) = -k_1 s(t) - k_0 sign(s) \tag{19} $$

The control law given in (17), ensures that the signal $s(t)$, and by extension the error signals $e(t)$ and $\dot{e}(t)$ are bounded and converge to zero as t goes to infinity. Unfortunately, it is impossible to implement the obtained control law due to the fact that the values of B_f, $u^f(t)$, and B_h are unknown, therefore we propose an adaptive control law based on the previously defined fractional sliding surface. In the following we propose an adaptive control law based on the previously defined fractional sliding surface.

3.3 Adaptive controller design

The adaptive control scheme is constructed as follows

$$ v(t) = E(t)^T\,\theta \tag{20} $$

Where $\theta = [\theta_1, \theta_2, \theta_3]$ denotes the vector of the adaptive parameters and $E(t)$ represents the regression vector, for this controller we take it as a combination some fractional derivatives of the error signal, we propose the following expression

$$ E(t) = [e(t), \mathcal{D}^{1-\alpha} e(t), \dot{e}(t)] \tag{21} $$

Let us define the following quadratic cost function

$$ J(\theta) = \tfrac{1}{2}\overline{b_h} e_v(t) \tag{22} $$

Where $e_v(t)$ represents the error between the optimal controller value $v^*(t)$ and the actual value $v(t)$

$$\begin{aligned} e_v(t) &= v^*(t) - v(t) \\ &= v^*(t) - \theta^T E(t) \end{aligned} \qquad (23)$$

Using the gradient descent method, we get

$$\dot{\theta}(t) = -\gamma \nabla_\theta J(\theta) \qquad (24)$$

Where γ denote the adaptation gain, after careful substitutions we finally obtain the following controller parameter update law

$$\begin{aligned} \dot{\theta}(t) = \gamma E(t) \Big(\mathcal{D}^{1-\alpha}\big(\dot{s}(t) + k_1 s(t)\big) \\ + k_0 sign(s)\Big) \\ - \gamma\sigma\theta(t) \end{aligned} \qquad (25)$$

To ensure the robustness of the adaptive control law we introduced a σ-modification term [22]. A comprehensive study and analysis of the stability of the proposed controller (19) and its parameter update law (23) is presented in [23].

4. SIMULATION RESULTS

To evaluate the performance of the adaptive controller in accommodating for different actuator failure patterns, we will perform multiple simulations on the dual ball beam system subject to actuator failure, the values of the system parameters used for the simulation are taken from (ref): $m = 0.12\,kg$, $r = 0.01\,m$, $g = 9.807\,m/s^2$, $d = 0.05\,m$, $L = 0.45\,m$, $J = 1\,Nms^2/rad$ the simulations will be performed in MATLAB/Simulink using the fractional order operator block from the FOMCOM toolbox [24]. The controller parameters are given in table 1

Table 1Controller parameter values for simulations

Parameter	Value
k_1	200
k_0	200
γ	10
σ	10^{-3}
λ	2

Scenario 1: The ball is placed initially at a distance $y(0) = 0.1m$, the objective is to position the ball at the center of the beam, the simulation is performed for $30s$, the second motor remains intact, and the first actuator will suffer a lock-in-place failure at $t = 10s$, with value $u_1{}^f(t) = 1$. The results of the simulation are displayed in Fig. 2 and Fig. 3, the position of the ball on the beam is given in Fig. 2, we simulated for different values of α in order to show the influence of the fractional order on the response of the controller under actuator faults, as can be observed from Fig. 3 for all the values of α the proposed controller manages to eliminate the effect of the stuck motor, and was able to keep the ball at the origin. At the instant of failure a jump occurs, however it is immediately handled with the difference in the response for different values of α for example for the higher values of α the controller manages to accommodate for the failure but with a larger deviation, and for lower values of α the jump is less significant but the response time is slower. As for Fig. 3 representing the inputs of both motors, we can clearly observe that when the first motor fails at $t = 10s$ the second healthy actuator will swiftly counteracts its effect by matching the failure value.

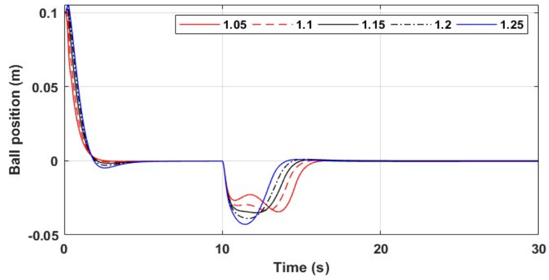

Fig. 2Ball position for failure scenario 1

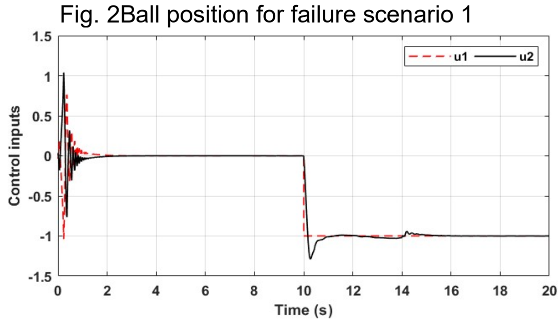

Fig. 3 Control inputs for failure scenario 1

Scenario 2: We position the ball initially at distance $y(0) = 0.05m$ from the origin, the objective is to center the ball on the beam $y_r(0) = 0$, we perform the simulation for 30 seconds, the second actuator is healthy throughout the simulation, and the first actuator suffers from a failure at instant $t = 10\,s$ and starts oscillating uncontrollably following the pattern $u_1{}^f(t) = 1 + 0.5sin(0.5t) - 0.5cos(t)$. The results of the simulation for the second failure scenario are presented in Fig. 4 and Fig. 5. From the onset we can clearly see that the response and ability of the controller to compensate for the faulty actuator corroborate with the results obtained in the first simulation; the controller manages to keep the position of the ball very close to the origin of the beam, it can be seen from Fig. 4 that for lower values of α, the controller provides much better tracking performances, while for higher values of α the controller still manages to keep the position close to zero although with slight oscillations.

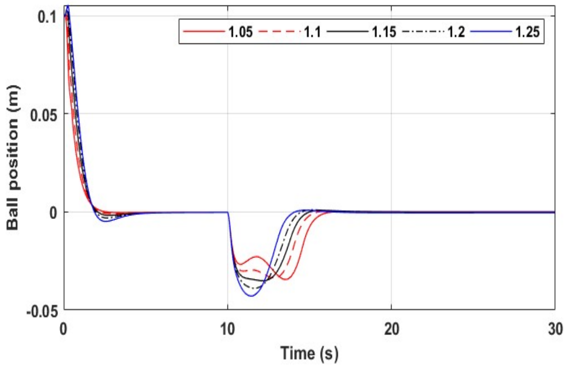

Fig. 4Ball position for failure scenario 2

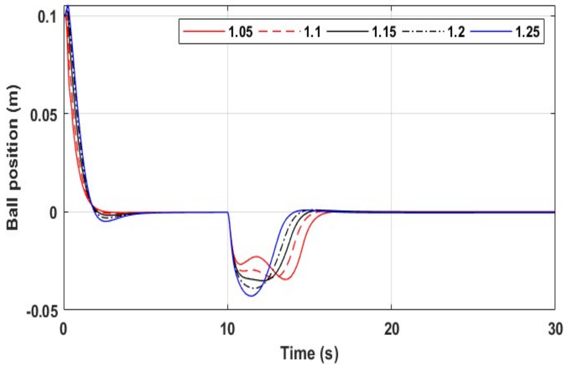

Fig. 5 Control inputs for failure scenario 2

It is clear from simulation results of both failure cases on the dual ball beam system that the proposed controller (19) with parameter update law (24) successfully achieved the prescribed control objective consisting of output tracking in the presence of actuator failures, it is also evident that the choice of fractional order α as a tuning parameter plays an important role in shaping the response of failure accommodation according to the desired dynamics, as concluded in preceding works [25-26].

5. CONCLUSION

We have successfully implemented a fractional adaptive controller for a dual ball-beam system with redundant motors, the proposed control strategy can accommodate for actuator failures that might occur on the motor function. Simulation results with different scenarios of actuators' failures illustrate the efficiency of the proposed controller. Our next works and research will targeted towards developing fractional order fault tolerant control strategies for other classes of systems such as MIMO systems, and include a more general class of actuator failures.

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