Stability Analysis and Controller Design of Networked Active Vehicle Suspension System Subject to Delays and Packet Losses

Nadjib Ben merabet(1)*, Abdelkrim Mansouria(1), Mohamed Rouamel(1) , Fayçal Bourahala(1)
(1) Department of Electrical Engineering, LAS Laboratory, 20 Août 1955 University, Skikda, Algeria.
*Corresponding author nadjibbenmerabet1@gmail.com

Abstract: In this paper, the stability analysis and control design of networked control systems (NCS) subject to network induced delay and packet dropouts, the study is aiming to maximizing the allowable upper bound of network induced delay together with the number of consecutive packet dropouts, The analysis method utilizes a new Lyapunov-Krasovskii (LKF) functional to provide less conservative results. The delay dependent stability criteria based on improved Wirtinger inequality and reciprocally convex combination inequality are derived in terms of linear matrix inequalities (LMI).Finally, numerical examples are given to show the effectiveness of the proposed analysis method compared to other existing methods.

Keywords: Network Control Systems (NCS), Lyapunov-Krasovskii Functional (LKF), Linear Matrix Inequalities (LMI)

1. INTRODUCTION

Networked control systems (NCSs) are spatially distributed systems in which all the components of the feedback control loop as the sensors, actuators, and controllers are connected through a communication network. In the last decade, the networked control systems NCSs have receive a growing interest due to the multiple advantages they offer, such as reduced installation costs, better maintainability, and greater flexibility. On the other hand, the use of the communication channel to interconnect the control loop components causes several problems and constraints such as network-induced delays, packet loss, quantization errors. These imperfections are responsible to degrade the system performances and even to make the considered NCS unstable. That’s why it is necessary to take them into account in the stability analysis of NCSs.

The recent researches are focusing on studies of the stability using the Lyapunov Krasovskii functional to derive the stability condition which takes into account the maximum information of negative network-induced effects which leads often to less conservative than imperfections-independent ones. Based on the Lyapunov-Krasovskii theorem the authors in [1] propose a new Lyapunov–Krasovskii function using the information of both the lower and upper bounds of the time-varying network-induced delay to drive a new delay-dependent H∞ stabilization criterion. The authors in [2, 3] focused in the construction of an appropriate LKFs with double, triple and quadruple-integral terms to provide larger delay bounds. NCSs developers used several new Lyapunov-Krasovskii functions to derive the NCSs stability conditions, but when they used the LKFs to include more than a single integral term, they fell into the problem of estimation of the LKFs time derivative (how getting the exact estimation of LKFs time derivative). This problem set an attraction to the mathematics and NCSs researchers to derive the good estimation. The papers in [4], [5,6,7,8]use Jensen inequality to estimate simple and double integral terms of Lyapunov time derivative while, the authors in [9, 10] used Wirtinger-based inequalities to estimate the cross term (single, double and triple integrals forms) in deriving process, the authors in [9,11] use free-weighting matrix to handle the cross-terms in the derivative of LKFs.

In this paper, we present a new study on the stability analysis and controller design of a network active vehicle suspension subject to delays and packet losses, using Lyapunov-Krasovskii functional to get the information of the effect of the network induced delay on the system. The resulting cross terms are estimated using a novel integral inequalities and free weighting matrices based on the Leibniz-Newton formula are introduced using null terms. Furthermore, to obtain less conservatism, Finsler's lemma is used to relax the LMI's stability conditions.

The resolution of the obtained LMI-based stability conditions allows us to obtain a maximum allowable upper network-induced delay bound and the a number of packet losses by. In order to demonstrate the feasibility of the proposed method, we provide a numerical example representing linear system controlled over network. Finally, a comparison with other previous results in terms of conservativeness and effectiveness is given.

2. PRELIMINARIES

Let us consider the modeling of active vehicle suspension as class of continuous-time linear systems described by the following state space representation.

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_W\,\omega(t) \\ y(t) = Cx(t) + Du(t) + D_W\,\omega(t) \end{cases} \qquad (2.1)$$

Where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are state and control vectors respectively $y(t) \in \mathbb{R}^p$ is the output vector, $w(t) \in \mathbb{R}^q$ is a vector of external disturbances belonging to $L_2[0, \infty]$, A, B, B_w, C, D and D_w are real constant matrices describe in the previous chapter and recalled as [18]:

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & -\frac{C_s}{m_s} & 0 & \frac{C_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{C_s}{m_u} & -\frac{k_u}{m_u} & -\frac{C_s+C_u}{m_u} \end{bmatrix}, B = \begin{bmatrix} 0 \\ \frac{1}{m_s} \\ 0 \\ -\frac{1}{m_u} \end{bmatrix}, B_\omega = \begin{bmatrix} 0 \\ 0 \\ -1 \\ \frac{C_u}{m_u} \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad D = 0, \quad D_w = 0.$$

Our goal is to investigate the control of uncertain and disturbed systems (1), according to the networked control scheme presented in Figure 1. In this context, the following assumptions are considered

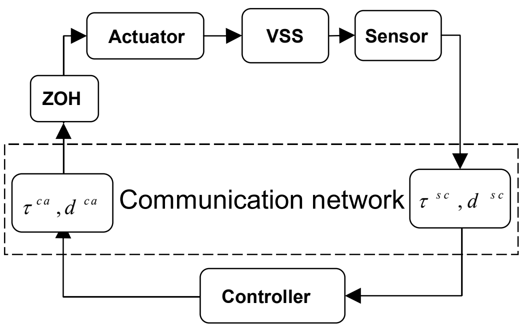

Fig.1. Schematic overview of the networked control system

Assumption 1: The Zero Order Hold (ZOH), controller and actuator are event driven, while the sensor is clock driven with a fixed sampling period.

Assumption 2: Suppose all state variables are available and transmitted to the controller through the network in single packets.
The sampling instants of the state variables are transmitted from the sensor to the ZOH are designated by the set $[t_0, t_1, t_2 \; . . . , t_k]$. During the transmission process, there are two types of delays /packet dropout when data is transmitted: delay/packet loss from sensor-to-controller τ_{sc}^k delay/packet loss from controller-to-actuator τ_{ca}^k, $\forall k \in \mathbb{N}$. The delays and packet loss in NCS can be described as:

$$\tau_k = \tau_{sc} + \tau_{ca}, \qquad d_k = d_{sc} + d_{ca}$$

By considering the global packet loss in the round transition as a kind of delays we can modeled as:

$$\tau_{loss} = (d_k + 1) \times h.$$

Where $d(t)$ the number of packets loss and h is the sampling time, and the number of packet loss can be denoted by:

$$d_k = \frac{t_{k+1} - t_k}{h} - 1$$

Assumption3: The delay induced by the network τ_k is bounded, which satisfies: $0 \le \tau_m \le \tau_k \le \tau_M$ where τ_m and τ_M are constants, the ZOH keeps the control signal during the interval $[t_k + \iota_k, t_{k+1} + \iota_{k+1})$, until a new signal arrives.
Based on the previous assumption, for the stabilization of linear system (2.1) we consider:
$\forall t \in [t_k + \iota_k, t_{k+1} + \iota_{k+1})$, the sampled-data state feedback control law described as follows:

$$\hat{u}(t) = Kx(t_k) \qquad (2.2)$$

Where K is the control gain matrix.

Based on the definition of the updating intervals of the ZOH, let's define:

$$\tau(t) = t - \iota_k, \qquad \forall t \in [t_k + \iota_k, t_{k+1} + \iota_{k+1}) \qquad (2.3)$$

Therefore, the control law (2) can be rewritten as:

$$u(t)=Kx(t-\tau(t)),\quad t\in[t_k+\iota_k,t_{k+1}+\iota_{k+1})\ (2.4)$$

According to the previous assumptions, the delay induced by the network can be defined as:

$$0\le\tau_1=\tau_m\le\tau(t)=t-t_k\le\tau_2=\tau_M+(d(t)+1)h,$$
$$\dot{\tau}(t)=1 \qquad (2.5)$$

By substituting the state feedback controller (2.4) in (2.1), closed loop dynamics as:

$$\begin{cases}\dot{x}(t)=Ax(t)+BKx(t-\tau(t))+B_w\omega(t)\\ y(t)=Cx(t)+DKx(t-\tau(t))+D_w\omega(t)\\ x(t)=\Phi(t),\ \forall t\in[-\tau_2,0]\end{cases}\qquad (2.6)$$

Where, $\Phi(t)$ are the initial state conditions.

The main goal of this work is to stabilize the closed-loop vehicle suspension (2.6) under the maximum available upper bound of $\tau(t)$, denoted $\mathrm{maub}(\tau_2)$, for various values of τ_1. To achieve this goal, some lemmas are useful:

Lemma 1 [24]: Let Ξ_1, Ξ_2 and Θ constant Matrices of appropriate dimensions and $0\le\eta_m\le\tau(t)\le\eta_M$, then the inequality

$$(\eta_M-\tau(t))\Xi_1+(\tau(t)-\eta_m)\Xi_2+\Theta<0 \quad (2.8)$$

Hold if and only if:

$$(\eta_M-\eta_m)\Xi_1+\Theta<0,\ (\eta_M-\eta_m)\Xi_2+\Theta<0 \quad (2.9)$$

Lemma 2 [25]: For any positive matrix R, real scalars a and b satisfying $a<b$, the inequalities (2.10), hold for any differentiable function $[a,b]\to R^n$ .x:

$$-\int_a^b\dot{x}^T(s)R\dot{x}(s)ds\le\frac{-1}{b-a}\begin{bmatrix}\psi_1\\ \psi_2\end{bmatrix}^T\begin{bmatrix}R & 0\\ * & 3R\end{bmatrix}\begin{bmatrix}\psi_1\\ \psi_2\end{bmatrix}\quad(2.10)$$

With:
$$\psi_1=x(b)-x(a),$$
$$\psi_2=x(b)+x(a)-\frac{2}{b-a}\int_a^b x(s)ds,$$

Lemma 3 [26]: For any vectors x_1, x_2 constant matrices W, N_i, $(i\in I_4)$, and real constant scales $\alpha>0$ and $\beta>0$ satisfying $\alpha+\beta=1$, the following inequality holds:

$$-\frac{1}{\alpha}x_1^TN_1x_1-\frac{\beta}{\alpha}x_1^TN_3x_1-\frac{1}{\beta}x_2^TN_2x_2-\frac{\alpha}{\beta}x_2^TN_4x_2$$
$$\le-\begin{bmatrix}x_1\\ x_2\end{bmatrix}^T\begin{bmatrix}N_1 & W\\ W^T & N_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Subject to:
$$\begin{bmatrix}N_1+N_3 & W\\ W^T & N_2+N_4\end{bmatrix}>0$$

Lemma4 [27]: Let $\xi\in R^n, G\in\mathcal{R}^{m\times n}$ and $Q=Q^T\in R^{n\times n}$ such that $\mathrm{rank}(G)<n$. The following statements are equivalent:

$$\xi^TQ\xi<0,\quad\forall\xi\in\{\xi\in\mathcal{R}^n:\xi\ne0,G\xi=0\}\quad(2.11)$$
$$\exists R\in\mathcal{R}^{n\times m}:Q+H_e(RG)\prec0\quad(2.12)$$

3. Main results: The main contribution of this work is to establish new robust stability conditions in form of LMI-based conditions, so that the networked active vehicle suspension (3.6) is asymptotically stable.

3.1.1 Stability analysis for VSS:

In this section, a novel networked active vehicle suspension stability conditions are developed based on the Lyapunov approach. So, new LKFs is proposed and the time derivative of LKF is bounded using recent lemma. Then, the main result of is addressed in the following Theorem:

Theorem 3.1: for given scalars $\tau_1>0,\tau_2>0$ such that $\tau_1\le\tau(t)\le\tau_2$ with K the gain of the state feedback matrix, the NCS model (2.6) with a delay interval induced by the network is globally asymptotically stable if there are positive definite $P=P^T, R_i=R_i^T, Q_i=Q_i^T\ (i\in\mathcal{I}_2)$, $S=S^T, (\in\mathbb{R}^{3n\times3n}), M=M^T, (\in\mathbb{R}^{4n\times4n})\ L=L^T$, matrices, real matrices $\mathcal{T}(\in\mathbb{R}^{8n\times n})$ and $\mathcal{W}$ such that the following conditions hold for:

$$\begin{bmatrix}\begin{matrix}R_2 & 0\\ 0 & 3R_2\end{matrix} & \mathcal{W}\\ \mathcal{W}^T & \begin{matrix}R_2 & 0\\ 0 & 3R_2\end{matrix}\end{bmatrix}>0\quad(3.1)$$

$$\Gamma^q=\begin{bmatrix}\sum_{i=1}^{3}\Phi_i+\mathcal{H}_e(\mathcal{T}\mathcal{G}) & \mathcal{D}^T\\ * & -I\end{bmatrix}<0,\quad(3.2)$$

With:

$$\mathcal{G}=\begin{bmatrix}A & 0 & BK & 0 & 0 & 0 & 0 & -I & B_w\end{bmatrix},$$

$$\mathcal{D}=\begin{bmatrix}C & 0 & DK & 0 & 0 & 0 & 0 & 0 & D_W\end{bmatrix},$$

$$\Phi_1=\begin{bmatrix}\Phi_2^{11} & -2R_1 & 0 & 0 & 6R_1 & 0 & 0 & P\\ * & \Phi_2^{22} & 0 & 0 & 6R_1 & 0 & 0 & 0\\ * & * & \Phi_2^{33} & 0 & 0 & 0 & 0 & 0\\ * & * & * & -Q_2 & 0 & 0 & 0 & 0\\ * & * & * & * & -12R_1 & 0 & 0 & 0\\ * & * & * & * & * & 0 & 0 & 0\\ * & * & * & * & * & * & 0 & 0\\ * & * & * & * & * & * & * & \Phi_2^{88}\end{bmatrix} \tag{3.3}$$

$$\Phi_2^{11}=Q_1-4R_1,\ \Phi_2^{22}=-Q_1-4R_1$$

$$\Phi_2^{33}=(1-\dot{\tau}(t))Q_2,\ \Phi_2^{8,8}=(\tau_1^2R_1+(\tau_2-\tau_1)^2R_2)$$

$$\Phi_2=\begin{bmatrix}\begin{bmatrix}(e_2-e_3)^T\\(e_2+e_3-2e_7)^T\end{bmatrix}^T\\ \begin{bmatrix}(e_3-e_4)^T\\(e_3+e_4-2e_8)^T\end{bmatrix}^T\end{bmatrix}\begin{bmatrix}\begin{bmatrix}R_3 & 0\\ * & 3R_3\end{bmatrix} & \begin{bmatrix}W_{11} & W_{12}\\W_{21} & W_{22}\end{bmatrix}\\ \begin{bmatrix}W_{11} & W_{12}\\W_{21} & W_{22}\end{bmatrix}^T & \begin{bmatrix}R_3 & 0\\ * & 3R_3\end{bmatrix}\end{bmatrix}\begin{bmatrix}\begin{bmatrix}(e_2-e_3)^T\\(e_2+e_3-2e_7)^T\end{bmatrix}^T\\ \begin{bmatrix}(e_3-e_4)^T\\(e_3+e_4-2e_8)^T\end{bmatrix}^T\end{bmatrix}^T \tag{3.4}$$

$$\Phi_1^1=(\tau_2-\tau_1)\left(\mathcal{H}_e\left(\begin{bmatrix}e_1\\e_3\end{bmatrix}^T\begin{bmatrix}N_{11} & N_{12}\\ * & N_{22}\end{bmatrix}\begin{bmatrix}e_5\\0\end{bmatrix}\right)+e_5N_2e_5^T\right)+\frac{1}{\tau_1}\begin{bmatrix}e_1\\e_3\end{bmatrix}^T\begin{bmatrix}-N_2 & N_2\\ * & -N_2\end{bmatrix}\begin{bmatrix}e_1\\e_3\end{bmatrix}^T$$
$$-(\tau_2-\tau_1)\begin{bmatrix}e_1\\e_2\\e_3\\e_4\end{bmatrix}^T\begin{bmatrix}M_{11} & M_{12} & M_{13} & M_{14}\\ * & M_{22} & M_{23} & M_{24}\\ * & * & * & M_{25}\\ * & * & * & M_{26}\end{bmatrix}\begin{bmatrix}e_1\\e_2\\e_3\\e_4\end{bmatrix} \tag{3.5}$$

$$\Phi_1^2=\frac{1}{\tau_2}\begin{bmatrix}e_1\\e_3\end{bmatrix}^T\begin{bmatrix}-N_2 & N_2\\ * & -N_2\end{bmatrix}\begin{bmatrix}e_1\\e_3\end{bmatrix}^T+(\tau_2-\tau_1)\begin{bmatrix}e_1\\e_2\\e_3\\e_4\end{bmatrix}^T\begin{bmatrix}M_{11} & M_{12} & M_{13} & M_{14}\\ * & M_{22} & M_{23} & M_{24}\\ * & * & * & M_{25}\\ * & * & * & M_{26}\end{bmatrix}\begin{bmatrix}e_1\\e_2\\e_3\\e_4\end{bmatrix}$$

Proof: Consider a Lyapunov-Krasovskii functional of the following form:

$$V(t)=\sum_{i=1}^{4}V_i(t) \tag{3.6}$$

Where:

$$V_1(t)=x^T(t)Px(t) \tag{3.7}$$

$$V_2(t)=\int_{t-\tau_1}^{t}x^T(s)Q_1x(s)ds+\int_{t-\tau_2}^{t-\tau(t)}x^T(s)Q_2x(s)ds+$$
$$\tau_1\int_{-\tau_1}^{0}\int_{t+\beta}^{t}\dot{x}^T(s)R_1\dot{x}(s)dsd\beta+(\tau_2-\tau_1)\int_{-\tau_2}^{-\tau_1}\int_{t+\beta}^{t}\dot{x}^T(s)R_2\dot{x}(s)dsd\beta \tag{3.8}$$

$$V_3(t)=(\tau_2-\tau(t))\left(\theta_1^T(t)S\theta_2(t)+\int_{t-\tau(t)}^{t}\dot{x}^T(s)L\dot{x}(s)ds\right)$$
$$+(\tau_2-\tau(t))(\tau(t)-\tau_1)\theta_2^T(t)M\theta_2(t) \tag{3.9}$$

$$\theta_1(t)=\begin{bmatrix}x^T(t) & x^T(t-\tau(t)) & \int_{t-\tau_1}^{t}x^T(s)ds\end{bmatrix}^T$$

$$\theta_2(t)=\begin{bmatrix}x^T(t) & x^T(t-\tau_1) & x^T(t-\tau(t)) & x^T(t-\tau_2) & \int_{t-\tau_1}^{t}x^T(s)ds\end{bmatrix}^T$$

The LKF candidate (3.6) is positive if P, $R_i\ (i\in I_2)$, Q_i, S, L and $M\ (i\in I_2)$ are all positive definite matrices. In this case, the networked control system (2.6) with network-induce delay (2.5) is asymptotically stable if:

$$\dot{V}(t)=\sum_{i=1}^{4}\dot{V}_i(t)<0 \tag{3.10}$$

Let us first define the following augmented vector:

$$\xi(t)=\begin{bmatrix}x^T(t) & x^T(t-\tau_1) & x^T(t-\tau(t)) & x^T(t-\tau_2) & V_1^T & \dots & V_4^T & \dot{x}^T(t)\end{bmatrix}^T \tag{3.11}$$

Where:

$$V_1=\frac{1}{\eta_m}\int_{t-\eta_m}^{t}x(s)ds,\ V_2=\frac{1}{\tau(t)}\int_{t-\tau(t)}^{t}x(s)ds,$$

$$V_3=\frac{1}{(\tau(t)-\eta_m)}\int_{t-\tau(t)}^{t-\eta_m}x(s)ds,$$

$$V_4=\frac{1}{\eta_m^2}\int_{-\eta_m}^{0}\int_{t+\beta}^{t}x(s)dsd\beta$$

The derivate of first term in (3.7) is given as follow:

$$\dot{V}_1(t)=H_e(x^T(t)P\dot{x}(t)) \tag{3.12}$$

Likewise, let consider the time derivative of second term (3.8) in (3.6)

$$\dot{V}_2(t)=\dot{x}^T(t)(\tau_1^2R_1+(\tau_2-\tau_1)^2R_2)\dot{x}(t)+x^T(t)Q_1x(t)$$
$$-x^T(t-\tau_1)Q_1x(t-\tau_1)+(1-\tau(t))Q_2x(t-\tau(t))$$
$$-x^T(t-\tau_2)Q_2x(t-\tau_2)-\tau_1\int_{t-\tau_1}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds$$
$$-(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\dot{x}^T(s)R_2\dot{x}(s)ds \tag{3.13}$$

Applying Lemma (2.3) to first integral terms in (3.13) yields:

$$-\tau_1\int_{t-\tau_1}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds\le-\xi^T(t)$$
$$\times\left(\begin{bmatrix}[e_1-e_2]^T\\ [e_1+e_2-2e_5]^T\end{bmatrix}^T\begin{bmatrix}R_1 & 0\\ * & 3R_1\end{bmatrix}\begin{bmatrix}[e_1-e_2]^T\\ [e_1+e_2-2e_5]^T\end{bmatrix}\right)\xi(t)$$

By verify that $\alpha+\beta=1$, the second integral terms of can be bounded with the help of Lemma 1 and lemma 2 as

$$-(\eta_M-\eta_m)\int_{t-\tau(t)}^{t-\tau_1}\dot{x}^T(s)R_2\dot{x}(s)ds-(\eta_M-\eta_m)\int_{t-\tau_2}^{t-\tau(t)}\dot{x}^T(s)R_2\dot{x}(s)ds$$

$$\leq -\zeta^T(t)\left(\frac{1}{\alpha}\begin{bmatrix}(e_2-e_3)^T\\(e_2+e_3-2e_7)^T\end{bmatrix}^T\begin{bmatrix}R_3 & 0\\ * & 3R_3\end{bmatrix}\begin{bmatrix}(e_2-e_3)^T\\(e_2+e_3-2e_7)^T\end{bmatrix}\right.$$
$$\left.+\frac{1}{\beta}\begin{bmatrix}(e_3-e_4)^T\\(e_3+e_4-2e_8)^T\end{bmatrix}^T\begin{bmatrix}R_3 & 0\\ * & 3R_3\end{bmatrix}\begin{bmatrix}(e_3-e_4)^T\\(e_3+e_4-2e_8)^T\end{bmatrix}\right)\zeta(t)$$

The inequality (3.11) can be written as:

$$\dot{V}_2(t)\leq \xi^T(t)\Phi_2\xi(t) \quad (3.14)$$

With Φ_2 is given in (3.4).

Continuing by the time derivative of the third term in (3.5):

$$\dot{V}_3(t)=-\left(\theta_1^T(t)S\theta_2(t)+\int_{t-\tau(t)}^{t}\dot{x}^T(s)L\dot{x}(s)ds\right)$$
$$+2(\tau_2-\tau(t))\left(\theta_1^T(t)S\dot{\theta}_1(t)+\begin{bmatrix}x(t)\\x(t-\tau(t))\end{bmatrix}^T\begin{bmatrix}-L & L\\ * & -L\end{bmatrix}\begin{bmatrix}x(t)\\x(t-\tau(t))\end{bmatrix}\right)$$
$$+(\tau_2-\tau(t))\theta_2^T(t)M\theta_2(t)-(\tau(t)-\tau_1)\theta_2^T(t)M\theta_2(t)$$
(3.15)

With: $\dot{\theta}_1(t)=\begin{bmatrix}x(t)\\(1-\dot{\tau}(t))x(t-\tau(t))\\x(t)-x(t-\tau_1)\end{bmatrix}$

The inequality (3.15) can be written as:

$$\dot{V}_3(t)\leq \xi^T(t)\Phi_3\xi(t) \quad (3.16)$$

With Φ_3 is given in (3.5)

Then, the time derivative of (3.13) can be rewriting as follow:

$$\dot{V}(t)\leq \zeta^T(t)\sum_{i=1}^{3}\Phi_i\zeta(t)<0$$

With $\Phi_i\ (i\in\mathcal{I}_3)$ is defined in theorem 3.1.

Note that the NCS in (2.6) can be rewritten as $\mathcal{G}\zeta(t)=0$ with $\mathcal{G}$ detailed in Theorem 1. Therefore, applying Finsler's Lemma the closed loop NCS with input delay in (2.5) is asymptotically stable if there exists $\mathcal{T}\in R^{12n\times n}$ such that:

$$\dot{V}(t)\leq \zeta^T(t)\sum_{i=1}^{3}\Phi_i+\mathcal{H}_e(\mathcal{T}\mathcal{G})\zeta(t)<0, \quad (3.17)$$

Which holds by convexity if (3.17) is satisfied.

3.1.1 Design of the sampled-data state feedback controller for NAVSS:

In this section, we try to solve the problem of stabilization and synthesis state feedback controller for NAVSS, so that the closed-loop NCS system (2.4) is asymptotically stable. Our result is summarized by the following Theorem.

Theorem 3.2: for given scalars $\tau_1>0, \tau_2>0$, the closed loop Networked VSS model (2.6) with a delay interval induced by the network is asymptotically stabilized if there are positivedefinite matrices $X=X^T, \bar{P}=\bar{P}^T, \bar{R}_i=\bar{R}_i^T, \bar{Q}_i=\bar{Q}_i^T\ (i\in\mathcal{I}_2)$, $\bar{S}=\bar{S}^T, (\in\mathbb{R}^{3n\times 3n}),\ \bar{M}=\bar{M}^T, (\in\mathbb{R}^{4n\times 4n})$, $\bar{L}=\bar{L}^T$, real matrices and $\mathcal{W}$ $(\in\mathbb{R}^{2n\times 2n}), F=KX$ such that the following conditions hold for:

$$\begin{bmatrix}\bar{R}_2 & 0 & \multicolumn{2}{c}{\bar{\mathcal{W}}}\\ 0 & 3\bar{R}_2 & & \\ \multicolumn{2}{c}{\bar{\mathcal{W}}^T} & \bar{R}_2 & 0\\ & & 0 & 3\bar{R}_2\end{bmatrix}>0,\ \bar{\Gamma}^q=\begin{bmatrix}\sum_{i=1}^{3}\bar{\Phi}_i+\mathcal{H}_e(\mathcal{X}) & \bar{\mathcal{D}}^T\\ * & -I\end{bmatrix}<0,$$

With:

$$\mathcal{X}=\begin{bmatrix}AX & 0 & BF & 0 & \dots & 0 & -X & B_w\\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0\\ \varepsilon_1 AX & 0 & \varepsilon_1 BF & 0 & \dots & 0 & -\varepsilon_1 X & \varepsilon_1 B_w\\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & 0\\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0\\ \varepsilon_2 AX & 0 & \varepsilon_2 BF & 0 & \dots & 0 & -\varepsilon_2 X & \varepsilon_2 B_w\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\in\mathbb{R}^{8n\times 8n},$$

$$\Phi_1=\begin{bmatrix}\bar{\Phi}_2^{11} & -2\bar{R}_1 & 0 & 0 & 6\bar{R}_1 & 0 & 0 & P\\ * & \bar{\Phi}_2^{22} & 0 & 0 & 6\bar{R}_1 & 0 & 0 & 0\\ * & * & \bar{\Phi}_2^{33} & 0 & 0 & 0 & 0 & 0\\ * & * & * & -\bar{Q}_2 & 0 & 0 & 0 & 0\\ * & * & * & * & -12\bar{R}_1 & 0 & 0 & 0\\ * & * & * & * & * & 0 & 0 & 0\\ * & * & * & * & * & * & 0 & 0\\ * & * & * & * & * & * & * & \bar{\Phi}_2^{8,8}\end{bmatrix}$$

$\bar{\Phi}_2^{11}=\bar{Q}_1-4\bar{R}_1,\ \bar{\Phi}_2^{22}=-\bar{Q}_1-4\bar{R}_1$

$\bar{\Phi}_2^{33}=(1-\dot{\tau}(t))Q_2,\ \bar{\Phi}_2^{8,8}=(\tau_1^2\bar{R}_1+(\tau_2-\tau_1)^2\bar{R}_2)$

(3.18)

$$\bar{\Phi}_2 = \begin{bmatrix} \begin{bmatrix} (e_2-e_3)^T \\ (e_2+e_3-2e_7)^T \end{bmatrix}^T \\ \begin{bmatrix} (e_3-e_4)^T \\ (e_3+e_4-2e_8)^T \end{bmatrix}^T \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \bar{R}_3 & 0 \\ * & 3\bar{R}_3 \end{bmatrix} & \begin{bmatrix} \bar{W}_{11} & \bar{W}_{12} \\ \bar{W}_{21} & \bar{W}_{22} \end{bmatrix} \\ \begin{bmatrix} \bar{W}_{11} & \bar{W}_{12} \\ \bar{W}_{21} & \bar{W}_{22} \end{bmatrix}^T & \begin{bmatrix} \bar{R}_3 & 0 \\ * & 3\bar{R}_3 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} (e_2-e_3)^T \\ (e_2+e_3-2e_7)^T \end{bmatrix}^T \\ \begin{bmatrix} (e_3-e_4)^T \\ (e_3+e_4-2e_8)^T \end{bmatrix}^T \end{bmatrix} \quad (3.19)$$

$$\bar{\Phi}_1^1 = (\tau_2-\tau_1)\left(\mathcal{H}_e\left(\begin{bmatrix} e_1 \\ e_3 \end{bmatrix}^T \begin{bmatrix} \bar{N}_{11} & \bar{N}_{12} \\ * & \bar{N}_{22} \end{bmatrix} \begin{bmatrix} e_5 \\ 0 \end{bmatrix} \right) + e_5 N_2 e_5^T \right)$$
$$+ \frac{1}{\tau_1} \begin{bmatrix} e_1 \\ e_3 \end{bmatrix}^T \begin{bmatrix} -\bar{N}_2 & \bar{N}_2 \\ * & -\bar{N}_2 \end{bmatrix} \begin{bmatrix} e_1 \\ e_3 \end{bmatrix}^T$$
$$-(\tau_2-\tau_1) \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}^T \begin{bmatrix} \bar{M}_{11} & \bar{M}_{12} & \bar{M}_{13} & \bar{M}_{14} \\ * & \bar{M}_{22} & \bar{M}_{23} & \bar{M}_{24} \\ * & * & * & \bar{M}_{25} \\ * & * & * & \bar{M}_{26} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \quad (3.20)$$

$$\Phi_1^2 = \frac{1}{\tau_2} \begin{bmatrix} e_1 \\ e_3 \end{bmatrix}^T \begin{bmatrix} -\bar{N}_2 & \bar{N}_2 \\ * & -\bar{N}_2 \end{bmatrix} \begin{bmatrix} e_1 \\ e_3 \end{bmatrix}^T + (\tau_2-\tau_1)$$
$$\times \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}^T \begin{bmatrix} \bar{M}_{11} & \bar{M}_{12} & \bar{M}_{13} & \bar{M}_{14} \\ * & \bar{M}_{22} & \bar{M}_{23} & \bar{M}_{24} \\ * & * & * & \bar{M}_{25} \\ * & * & * & \bar{M}_{26} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}$$

In this case, the stabilizing NCS controller gain matrix is recovered by the change of variable:

$$K = FX^{-1} \quad (3.21)$$

Proof: The conditions expressed in Theorem 1 are no more LMIs when the gain K in the product $\mathcal{TG}$ is unknown. To cope with this product, let X be an invertible matrix and choose:

$\mathcal{T} = \begin{bmatrix} X^{-T} & 0 & \varepsilon_1 X^{-T} & 0 & \ldots & 0 & \varepsilon_2 X^{-T} & 0 \end{bmatrix}^T \in \mathbb{R}^{8n\times n}$

with positive scalars ε_1 and ε_2 then, pre-and-post-multiplying the inequalities (3.4) by $D_X^8 = diag\underbrace{[X \;\; \ldots \;\; X]}_{k\ times}^T$, and its transpose with the change of variables.

$F = KX, \bar{P} = X^T P X, \bar{Q}_i = X^T Q_i X,$
$\bar{L} = X^T L X, \bar{R}_i = X^T R_i X, (i \in \mathcal{I}_2),$
$\bar{S} = D_X^3 S D_X^{3T}, \bar{M} = D_X^4 M D_X^{4T}$ and $\bar{\mathcal{N}} = D_X^2 \mathcal{N} D_X^{2T}$

We obtain the conditions expressed in Theorem 3.2.

3.2. Simulation of networked active vehicle suspension:

In this section, we provide simulation of the Networked AVSS, to illustrate the efficiency and the reduction of conservatism of the results proposed in Theorems 3.1 and 3.2 compared to previous relevant results. The objective is to compare the maximum allowable limits of the delay and packet dropouts, which guarantees the overall stability of the systems studied.

Part I: Stability analysis (when the control gain K known)

By given the Quarter-car model parameters listed in the following table:

Table 3.1 Quarter-car model parameters

Quarter-car parameters	k_s	m_s	k_u	c_s	c_u	m_u
	42720 N/m	973 kg	101115 N/m	1095 Ns/m	14.6 Ns/m	114 kg

To show the effectiveness of stability conditions proposed in Theorem 1 regards to the work in [1] with the control gain $K = 10^4 \times [-8.922 - 0.1447 - 3.6650.1491];$ respectively. The goal maximum allowable upper bound of the network-induced delay using the LMI proposed in [1] is obtained as $\tau_2 = 0.001s$ and using our LMI is $\tau_2 = 0.020s$ with upper bound of consecutive packet loss obtained $d_M = 3$ that ensures that the asymptotic stability of the closed-loop AVSS is asymptotically stable.

As one can notice, among the considered results in [1], the ones obtained from Theorem 3.1 always provide the biggest maximal allowable values of τ_2 for several values of τ_1 This confirms the significant conservatism reduction provided by the LMI-based stability conditions proposed in Theorem 3.1 regarding to all the other considered results. Fig. 3.1 shows the state trajectories of theconsidered closed-loop NCS, and Fig. 3.2 show the updating instans and intervals (set for the simulation purpose as a random signal in $[\tau_1, \tau_2]$ and updated at each sampling instant t_k). As expected from the obtained solution of Theorem 3.2, this NCS is found stable, which confirms the effectiveness of the proposed NCS.

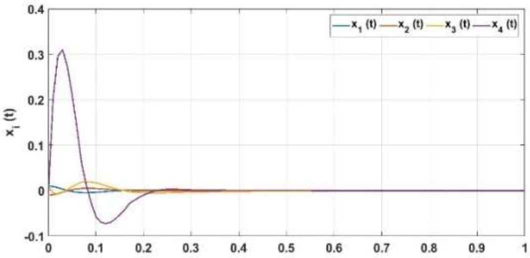

Fig.2. State trajectories of AVSS

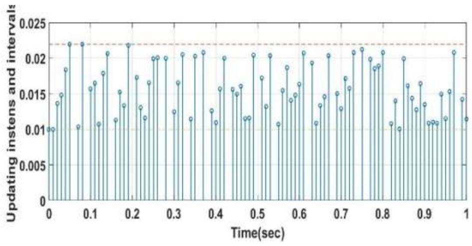

Fig.3. Updatinginstants and intervals of ZOH

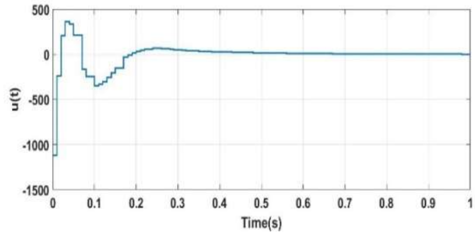

Fig.5. Commands trajectories of AVSS(Case 1).

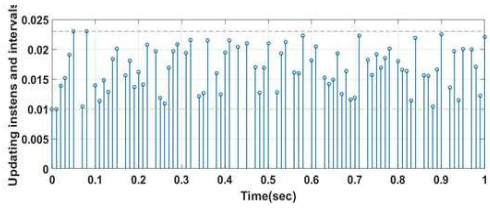

Fig.6.Updatinginstants and intervals of ZOH(Case 1).

Part II: Controller Design of Networked AVSS

Case 1: Using the parameter expressed in work [1]. For given $\tau_1 = \tau_m = 0.01,\ \varepsilon_1 = 0.0005$ and $\varepsilon_2 = 0.0002$, h=0.01, $\tau_M = 0.02$, and assume that there are 3 packet losses the condition of theorem 3.2 is solved and we obtain the state feedback sampled-data control law:

$$u(t) = 10^4 \times \left[-7.922\ -0.1047\ -3.225\ 0.1571\right] x(t-\tau(t))$$

The state trajectories of AVSS are given in Figure 3.3, the Commands trajectories in figure 3.4, the ZOH updating instants and intervals in figure 3.5, when the initial conditions are $x(t) = [-0.01 \quad 0.01 \quad -0.01 \quad 0.01]^T$.

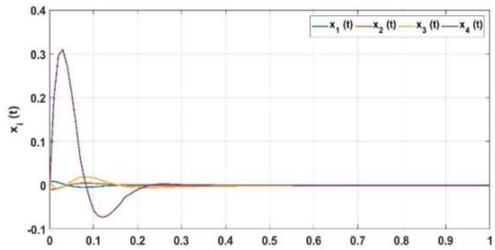

Fig.4. State trajectories of AVSS (Case 1)

Case 2: Setting new network parameter.

For given $\tau_1 = \tau_m = 0.01,\ \varepsilon_1 = 0.0005$ and $\varepsilon_2 = 0.0002$, h=0.01, and assume that there are 5 packet losses the maximum allowable value network-induced delay is $\tau_M = \tau_2 - (d_M + 1)h = 0.8 - (5+1)0.01 = 0.74s$ under the state feedback sampled-data control law:

$$u(t) = \left[-9.8741\ \ -0.3926\ \ 22.7563\ \ 0.5766\right] x(t-\tau(t))$$

With the above control law, the state trajectories of AVSS are given in Figure 3.6, the Commands trajectories in figure 3.8, the ZOH updating instants and intervals in figure 3.7, when the initial conditions are $x(t) = [-0.01 \quad 0.01 \quad -0.01 \quad 0.01]^T$. It can be seen that the state trajectories of system (3.6) with the commands signal trajectories of Figure 3.8 tends to zero, which demonstrates that the proposed state feedback controller stabilize the considered NCS (3.6).

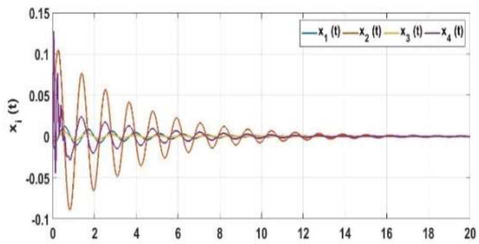

Fig.7.State trajectories of AVSS (Case2)

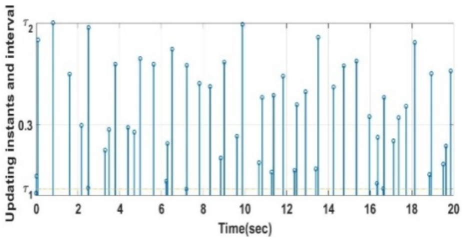

Fig.8. ZOH updating instants and intervals (Case 2)

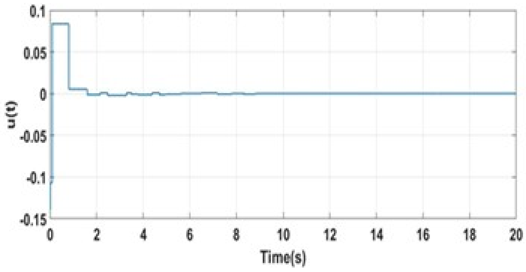

Fig.10.Commands trajectories of AVSS (Case 2)

3.3. Conclusion:

In this paper, we present a new method of stability and stabilization improvement of network control systems subject to network induced delay and packet dropouts. This method allows us to address the problem of stability analysis and control design for NCS in the form of LMIs, using Wirtinger inequality combined with improved reciprocally convexity, the resolution of this problem allows us to have more relaxation of the network induced delay. Applied on the above numerical example the obtained results are compared with those existing methods, which prove its efficiency in terms of conservativeness.

References

[1] H. Pan , Jing X , W. Sun , H. Gao , A bioinspired dynamics-based adaptive tracking control for nonlinear suspension systems, IEEE Trans. Control Syst. 26 (3) (2018) 903–914 .

[2] H. Pan , X Jing , W. Sun , H Gao , J, Yu , Finite-time stabilization for vehicle active suspension systems with hard constraints, IEEE Trans. Intell. Transp. Syst. 16 (5) (2015) 2663–2672.

[3] Y. Guan, Han Q.L, H. Yao, X. Ge, Robust event-triggered H∞ controller design for vehicle active suspension systems, Nonlinear Dyn.(2018). https:// doi.org/ 10.1007/ s11071- 018- 4381- 0.

[4] H.P. Wang, G.I.Y. Mustafa, Y. Tian, Model-free fractional-order sliding mode control for an active vehicle suspension system, Adv. Eng. Softw. 115 (2018) 452–461.

[5] V.S. Deshpande , P.D. Shendge , S.B. Phadke , Dual objective active suspension system based on a novel nonlinear disturbance compensator, Veh. Syst. Dyn. 54 (9) (2016) 1269–1290.

[6] R. Wang, H. Jing, F. Yan, H.R. Karimi, N. Chen, Optimization and finite-frequency H∞ control of active suspensions in in-wheel motor driven electric ground vehicles, J. Frankl. Inst. 352 (2) (2015) 468–484.

[7] G. Wang, C. Chen, S. Yu, Robust non-fragile finite-frequency H∞ static output-feedback control for active suspension systems, Mech. Syst. Signal Process. 91 (2017) 41–56.

[8] M. Rouamel, F. Bourahala, A. N.D. Lopes, N. Nafir, and K. Guelton.Mixed actual and memory data-based event-triggered H∞ control design for networked control system. In 4th IFAC Conference on Embedded Systems, Computational Intelligence and Telematics in Control. Proceedings. CESCIT'21, pages 1–6, 2021.

[9] No. Nafir, Z. Ahmida, K. Guelton, F. Bourahala, and M. Rouamel. Improved robust h-infinity stability analysis and stabilisation of uncertain and disturbed networked control systems with network induced delay and packet dropout. International journal of systems science 2021.

[10] M. Rouamel, S. Gherbi, and F. Bourahala. Robust stability and stabilization of networked control systems with stochastic time-varying network- induced delays. Transactions of the Institute of Measurement and control

[11] F. Bourahala, M. Rouamel, and K. Guelton. Improved robust hâ stability analysis and stabilization of uncertain systems with stochastic input time-varying delays. Optimal Control Applications and Methods, 2021.

[12] Wei Jin, Bin Tang, Jiali Qin, and Yun Zhang. Improvement on fuzzy-model-based stabilization of nonlinear-networked control systems. In Proceedings of the 33rd Chinese Control Conference, pages 5766–5773. IEEE, 2014.

[13] Jun Zhang and Da-Yong Luo. A new stability condition for networked control system with time-varying delay based on delay uneven-partitioning approach. In 2015 Chinese Automation Congress (CAC), pages 145–150. IEEE, 2015.

[14] Zhichen Li, Yan Bai, and Tianqi Li. Improved stability and stabilization design for networked control systems using new quadruple-integral functionals. ISA transactions, 63:170–181, 2016.

[15] Jiandong Sun and Jingping Jiang. Delay and data packet dropout separately related stability and state feedback stabilization of networked control systems. IETControl Theory & Applications, 7(3):333–342, 2013.

[16] Oh-Min Kwon, Myeong-Jin Park, Sang-Moon Lee, Ju H Park, and Eun-Jong Cha. Stability for neural networks with time-varying delays via some new approaches. IEEE transactions on neural networks and learning systems, 24(2):181–193, 2012.

[17] Pin-Lin Liu. Further results on delay-range-dependent stability with additive time varying delay systems. ISA transactions, 53(2):258–266, 2014.

[18] Wang, Gang, et al. "Event-triggered control for active vehicle suspension systems with network-induced delays." Journal of the FranklinInstitute 356.1(2019):147-172.

[19] Won Il Lee and Poogyeon Park. Second-order reciprocally convex approach to stability of systems with interval time-varying delays. Applied Mathematics and Computation, 229:245– 253, 2014.

[20] Meng Li, Yong Chen, Anjian Zhou, Wen He, and Xu Li. Adaptive tracking control for networked control systems of intelligent vehicle. Information Sciences, 503:493–507, 2019.

[21] Bing Li and Jun-feng WU. A new delay-dependent stability criteria for networked control systems. Journal of Theoretical and Applied Information Technology, 43(1):127–132, 2012.

[22] Shen, Y., Fei, M., Du, D., Peng, C., & Tian, Y.-C. (2018). Event-triggered robust H∞ control for uncertain networked control systems with time delay. Transactions of the Institute of Measurement and Control, 40(9), 2928–2947.

[23] H.P. Wang, G.I.Y. Mustafa, Y. Tian, Model-free fractional-order sliding mode control for an active vehicle suspension system, Adv. Eng. Softw. 115 (2018) 452–461.

[24] Meng Li, Yong Chen, Anjian Zhou, Wen He, and Xu Li. Adaptive tracking control for networked control systems of intelligent vehicle. Information Sciences, 503:493–507, 2019.

[25] Bing Li and Jun-feng WU. A new delay-dependent stability criteria for networked control systems. Journal of Theoretical and Applied Information Technology, 43(1):127–132, 2012.

[26] R. Skelon, T. Iwasaki, and K. Grigoriadis, A united algebra approach to linear controldesign, 1998.

[27] W. I. Lee, S. Y. Lee, and P. Park, Improved criteria on robust stability and h1 performancefor linear systems with interval time-varying delays via new triple integral functionals,Applied Mathematics and Computation, 243(2014), pp. 570{577.