Delay-Dependent Stability Improvement for Networked Control Systems:a Sampled Data Approach

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Abstract: This paper concerns the stability conditions and controller design for sampled-data networked control systems (NCSs) model subject to network communication delays, the main objective is to guaranty the maximum allowable upper bounds of network-induced time varying delays that keep the NCSs stable. First, Lyapunov-Krasovskii functional with simple and double-integral terms is constructed considering both upper and lower bounds of network delay. Then, less conservative Linear Matrix Inequalities (LMIs) stability conditions are established using null terms to introduce free weighting matrices based on the Leibniz-Newton formula. Furthermore, Finsler's lemma is used for the relaxation of the obtained LMI's stability conditions using slack decision variables. It is also used to decouple Lyapunov-Krasovskii matrices from the system ones. The application of the proposed approach for different NCSs gives higher upper delay bounds compared with other methods.

Keywords: Time delay, Stability, Linear Matrix Inequalities, Finsler lemma.

1. INTRODUCTION

The NCSs are decentralized systems where the control loop components (sensors, actuators, and controllers) are closed through single digital communication network channel. In recent years, NCSs has received considerable attention due to their many benefits in terms of cost, weight, installation and maintenance [6] over point to point wired conventional feedback control systems. In addition, due to the benefits of control over communication network and the development of the network access technologies, NCSs are much more employed in many real control systems such Wide area plant automation, intelligent transportation systems, remote surgery, robotics [4], distributed power systems and smart grids [1]. This led industrial societies and researchers to a growing interest in this field.However, due to the challenges created by the digital communication channel such, network-induced delays and packet losses, the system control performance may degrade and even lead to closed-loop instability [18].

Actually, many researchers in this field have made great progress to address the stability of NCS, wherein the main issue is to minimize the conservatism of the asymptotic stability criterion and guarantying at the same time maximum allowable delay bound(MADB) for NCSs. Among the most effective interesting methods to relax the stability conditions, free-weighting matrices are considered in[5],Delay-partitioning approach [8], relaxation matrices method[7], Wirtinger based Inequalitiesin[2].Morerecently,NCSs with stochastic time-varying network-induced delays are considered in [11,3] and event-triggered control has been considered to reduce sensors network resource consumption as power and network bandwidth in [12,14].

In this paper, we present a new study on the stability analysis and stabilization method for NCSs with a network-induced delay. To this end, Lyapunov-Krasovskii functional is used to get the information of the effect of networked delay on the system. The resulting cross terms are estimated using novel integral inequalities and free weighting matrices based on the Leibniz-Newton formula are introduced using null terms. Furthermore, in order to minimize the conservatism, Finsler's lemma is used to relax the LMI's stability conditions by adding slack decision variables. The resolution of the obtained LMI-based stability conditions allow us not only to obtain a maximum allowable upper network-induced delay bound by also derive robust state feedback controller gains. In order to demonstrate the feasibility of the proposed method, we provide two numerical examples representing linear system controlled over network. Finally, a comparison with others methods in terms of conservativeness and effectiveness is given.

Notations: In the text, in matrices denote bloc transpose quantities. For a matrix M of appropriate dimention, one denotes . $\mathbf{H}_e(M)=M+M^T$ .A finite set of r positive integers is denoted $I_r=\{1,\dots,r\}$ also, $\forall j\in I5$, the block entry matrices is expressed by* $e_j=\begin{bmatrix}0_{n\times(j-1)n} & I_{n\times n} & 0_{n\times(5n-j)n}\end{bmatrix}^T\in\mathcal{R}^{5n\times n}$ for example: $e_4=[0\quad 0\quad 0\quad I\quad 0]^T$.

2. System Model Description

Let consider the following NCS model

$$\begin{cases}\dot{x}(t)=Ax(t)+Bu(t)\\ x(0)=\phi(t),\, t\in[t_kh,t_{k+1}h)\end{cases}\qquad(1)$$

Where $x(t)\in\mathcal{R}^n$ state and $u(t)\in\mathcal{R}^m$ control vectors respectively A and B are constant matrices with appropriate dimensions $\phi(t)$ is the given function of initial conditions of the systems on the interval of time $t\in[t_kh,t_{k+1}h)$ where the sampling period is h. The following assumptions are adopted.

Assumption1: The zero-order holder (ZOH), the controller and the actuator are event-driven, whereasthe sensor is clock-driven with a fixed sampling period$h$.

Assumption2: Assume that all the state variables are available and transmitted to the controller through the network in single-packets. The sampling instants set of state variables are denoted as $[t_1h,t_2h,\dots,t_kh]$.

The sampling instants are transmitted throw a digital network. During the transmission process, there are two kinds of delays sensor-to-controller delay τ_{sc_k} andcontroller-to-actuator delay τ_{ca_k} .

The tow delays in this structure can be grouped together as $\tau_k=\tau_{sc_k}+\tau_{ca_k}$ .Therefore, the ZOH updating instants are denoted by $[t_1h+\tau_1,t_2h+\tau_2,\dots,t_kh+\tau_k)$.

Each control signal is maintained by ZOH and is only valid over time interval $[t_1h+\tau_1,t_2h+\tau_2,\dots,t_kh+\tau_k)$

Assumption3: The network-induced delay τ_k is bounded,and satisfies $0\le\tau_m\le\tau_k\le\tau_M$ where τ_m and τ_M are constants, the ZOH keeps the control signal through the interval $[t_1h+\tau_1,t_2h+\tau_2,\dots,t_kh+\tau_k)$ until the arrival of a new signalat $t_{\{k+1\}}h$.

Based on the previous assumptions, the state feedback control law can be described as:

$$u(t)=Kx(t_kh),\qquad t\in[t_kh+\tau_n,t_{k+1}h+\tau_{n+1})\quad(2)$$

Where K is the state feedback matrix gain.

Let us defining:

$$\tau(t)=t-t_kh,\, t\in[t_kh+\tau_k,t_{k+1}h+\tau_{n+1})\qquad(3)$$

Therefore, the control law u can be rewritten as:

$$u(t)=Kx(t-\tau(t)),\;\; t\in[t_kh+\tau_h,t_{k+1}h+\tau_{k+1})\quad(4)$$

According to the previous assumptions, the network-induced delay can be described as:

$$\begin{cases}0\le\tau_m\le\tau(t)=t-t_kh\le\tau_M,\, t\in[t_kh+\tau_k,t_{k+1}h+\tau_{k+1})\\ \dot{\tau}(t)=1,\;\; t\ne(t_kh+\tau_k)\end{cases}\qquad(5)$$

Substituting the state feedback controller into, we obtain the closed-loop dynamics as:

$$\begin{cases}\dot{x}(t)=Ax(t)+BKx(t-\tau(t)),\\ x(t)=\phi(t),\;\; t\in[t_kh+\tau_k,t_{k+1}h+\tau_{k+1}).\end{cases}\qquad(6)$$

Based on Lyapunov-Krasovskii Functional, our idea is to suggest LMI's conditions to study the stability and stabilization of NCS described in. To achieve this goal, some lemmas are useful:

Lemma1[13]: For any constant matrix scalars $\tau_1\le\tau(t)\le\tau_2$ and vector function $\dot{x}:[-\tau_2,-\tau_1]\to\mathcal{R}^n$ such that the following integration is well defined, the following inequality holds:

$$\begin{aligned}&-(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\dot{x}(s)^TQ\dot{x}(s)ds\le\\ &-(x(t-\tau_1)-x(t-\tau(t))Q(x(t-\tau_1)-x(t-\tau(t))^T\\ &-(x(t-\tau(t)-x(t-\tau_2))Q(x(t-\tau(t)-x(t-\tau_2))^T\\ &-\alpha(x(t-\tau_1)-x(t-\tau(t))Q(x(t-\tau_1-x(t-\tau(t))^T\\ &-\beta(x(t-\tau(t))-x(t-\tau_2))Q(x(t-\tau(t)-x(t-\tau_2))^T\end{aligned}\qquad(7)$$

$$Where:\quad \alpha=\frac{(\tau_2-\tau(t))}{\tau_2-\tau_1} and\ \beta=\frac{(\tau(t)-\tau_1)}{\tau_2-\tau_1}$$

Lemma2 [15]: Let $\xi \in R^n, G \in \mathcal{R}^{m\times n}$ and $Q = QT \in Rn \times n$ such that $\mathrm{rank}(G) < n$. The following statements are equivalent:

$$\xi^T Q \xi < 0, \quad \forall \xi \in \{\xi \in \mathcal{R}^n : \xi \neq 0, G\xi = 0\} \quad (8)$$

$$\exists R \in \mathcal{R}^{n\times m} : Q + H_e(RG) \prec 0 \quad (9)$$

Lemma3[17]: Let $\varphi 1, \varphi 2$ and Ω constant matrices of appropriate dimensions, and $\tau_1 \le \tau(t) \le \tau_2$ then the inequality

$$(\tau(t) - \tau_1)\varphi_1 + (\tau_2 - \tau(t))\varphi_2 + \Omega \le 0 \quad (10)$$

Hold if and only if:

$$(\tau_2 - \tau_1)\varphi_1 + \Omega \le 0 \quad \text{and} \quad (\tau_2 - \tau_1)\varphi_2 + \Omega \le 0 \quad (11)$$

3. Main results

In the following, a new NCSs stability criteria based on an appropriate LKF will be developed. For the convenience of narration, we define $\tau_1 = \tau_m$ and $\tau_2 = \tau_M + h$.

3.1. Stability analysis for NCS

Let us first focus on the stability analysis of NCS, our result is proposed in the following theorem

Theorem 1: Let $i \in I_2$ and $j \in I_3$. For given scalars $\tau_i > 0$ suchthat $\tau_1 \le \tau(t) \le \tau_2$ and feedback controller gain matrix K, the NCS model (6) with network-induced delay (3) is GASif there exist symmetric positive definite matrices $P, S, Q_i, R_j (in \mathcal{R}^{n\times n})$ and variables N_1 and $N_2 (in \mathcal{R}^{n\times n})$ and real matrix $T (in \mathcal{R}^{5n\times n})$ such that the following LMIsconditions hold for both $q = 1$ and $q = 2$:

$$\begin{bmatrix} \sum_{i=1}^{3} \Phi_i + \Xi^q + H_e(TG) & (\tau_2 - \tau_1) N^q \\ * & -(\tau_2 - \tau_1) S \end{bmatrix} < 0 \quad (12)$$

Where

$$G = \begin{bmatrix} A & 0 & BK & 0 & -I \end{bmatrix} \quad (13)$$

$$\Phi_1 = H_e(e_1 P e_5^T) + e_1(Q_1 + Q_2)e_1^T - e_2 Q_1 e_2^T - e_4 Q_2 e_4^T \quad (14)$$

$$\Phi_2 = (\tau_2 - \tau_1) e_5 S_1 e_5^T \quad (15)$$

$$\Phi_3 = H_e\left(N^1 (e_3 - e_4)^T + N^2 (e_2 - e_3)^T\right) \quad (16)$$

With

$$\begin{aligned} \Xi^1 = {} & e_5\left(\tau_1^2 R_1 + \tau_2^2 R_2 + (\tau_2 - \tau_1)^2 R_3\right)e_5^T - (e_1 - e_2)R_1(e_1 - e_2)^T \\ & -(e_2 - e_3)R_2(e_2 - e_3)^T - (e_3 - e_4)R_2(e_3 - e_4)^T - (e_3 - e_4)R_2(e_3 - e_4)^T \\ & -\frac{\tau_1}{\tau_2}(e_1 - e_3)R_3(e_1 - e_3)^T - (e_1 - e_3)R_3(e_1 - e_3)^T - (e_3 - e_4)R_3(e_3 - e_4)^T \quad (17) \\ & -\frac{\tau_2 - \tau_1}{\tau_2}(e_3 - e_4)R_3(e_3 - e_4)^T \end{aligned}$$

$$\begin{aligned} \Xi^2 = {} & e_5\left(\tau_1^2 R_1 + \tau_2^2 R_2 + (\tau_2 - \tau_1)^2 R_3\right)e_5^T - (e_1 - e_2)R_1(e_1 - e_2)^T \\ & -(e_2 - e_3)R_2(e_2 - e_3)^T - (e_3 - e_4)R_2(e_3 - e_4)^T - (e_1 - e_3)R_3(e_1 - e_3)^T \quad (18) \\ & -(e_3 - e_4)R_3(e_3 - e_4)^T - (e_2 - e_3)R_2(e_2 - e_3)^T - (e_1 - e_3)R_3(e_1 - e_3)^T. \end{aligned}$$

Proof1: Consider the following LKF candidate:

$$V(t) = V_1(t) + V_2(t) + V_3(t) \quad (19)$$

$$V_1(t) = x(t)^T(t) P x(t) + \int_{t-\tau_1}^{t} x(s)^T Q_1 x(s) ds + \int_{t-\tau_2}^{t} x(s)^T Q_2 x(s) ds \quad (20)$$

$$\begin{aligned} V_2(t) = {} & \tau_1 \int_{-\tau_1}^{0} \int_{t+s}^{t} \dot{x}(s)^T R_1 \dot{x}(s) ds dv \\ & + \tau_2 \int_{-\tau_2}^{0} \int_{t+s}^{t} \dot{x}(s)^T R_2 \dot{x}(s) ds dv \\ & + (\tau_2 - \tau_1) \int_{-\tau_2}^{-\tau_1} \int_{t+s}^{t} \dot{x}(s)^T R_3 \dot{x}(s) ds dv \quad (21) \end{aligned}$$

$$V_3(t) = \int_{-\tau_2}^{-\tau_1} \int_{t+s}^{t} \dot{x}(s)^T S \dot{x}(s) ds dv \quad (22)$$

The LKF candidate is positive if the given matrices $P, R_1, R_2, R_3, Q_1, Q_2$ and S are positive definite matrices. Then, the Networked control system with network-induced delay is GAS if:

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) < 0 \quad (23)$$

Let us first focus on $V_1(t)$ from (20) one has:

$$\begin{aligned} \dot{V}_1(t) &= H_e(x(t)^T P \dot{x}(t)) + x^T(t)(Q_1 + Q_2)x(t) \\ & \quad - x^T(t - \tau_1) Q_1 x(t - \tau_1) - x^T(t - \tau_2) Q_2 x(t - \tau_2) \\ &= \zeta^T(t) \Phi_1 \zeta(t) \quad (24) \end{aligned}$$

With $\Phi 1$ given in (14) and

$$\zeta(t) = \begin{bmatrix} x^T(t) & x^T(t - \tau_1) & x^T(t - \tau(t)) & x^T(t - \tau_2) & \dot{x}^T(t) \end{bmatrix}^T \quad (25)$$

Now, let us focus on the time derivative of (21), one has:

$$\dot{V}_2(t) = \dot{x}^T(t)\left(\tau_1^2 R_1 + \tau_2^2 R_2 + (\tau_2-\tau_1)^2 R_2\right)\dot{x}(t) - \tau_1\int_{t-\tau_1}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds - \tau_2\int_{t-\tau_2}^{t}\dot{x}^T(s)R_2\dot{x}(s)ds - (\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\dot{x}^T(s)R_3\dot{x}(s)ds \quad (26)$$

Applying lemma1, it yields:

$$-\tau_1\int_{t-\tau_1}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds \le -(e_1-e_2)R_1(e_1-e_2)^T + (e_1+e_2-2e_5)R_1(e_1+e_2-2e_5)^T \quad (27)$$

$$\begin{aligned}(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\dot{x}^T(s)R_2\dot{x}(s)ds \le\\ -(x(t-\tau_1)-x(t-\tau(t))R_2(x(t-\tau_1)-x(t-\tau(t)))^T\\ -(x(t-\tau(t)-x(t-\tau_2))R_2(x(t-\tau(t))-x(t-\tau_2))^T\\ -\alpha_1(x(t-\tau_1)-x(t-\tau(t))R_2(x(t-\tau_1)-x(t-\tau(t)))^T\\ -\beta_1(x(t-\tau(t))-x(t-\tau_2))R_2(x(t-\tau(t))-x(t-\tau_2))^T\end{aligned} \quad (28)$$

$$\begin{aligned}(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\dot{x}^T(s)R_2\dot{x}(s)ds \le\\ -(x(t-\tau_1)-x(t-\tau(t))R_2(x(t-\tau_1)-x(t-\tau(t)))^T\\ -(x(t-\tau(t)-x(t-\tau_2))R_2(x(t-\tau(t))-x(t-\tau_2))^T\\ -\alpha_1(x(t-\tau_1)-x(t-\tau(t))R_2(x(t-\tau_1)-x(t-\tau(t)))^T\\ -\beta_1(x(t-\tau(t))-x(t-\tau_2))R_2(x(t-\tau(t))-x(t-\tau_2))^T\end{aligned}$$

$$Where\ \ \alpha_1 = \frac{\tau_2-\tau(t)}{\tau_2-\tau_1} and\ \ \beta_1 = \frac{(\tau(t)-\tau_1)}{\tau_2-\tau_1}$$

$$\begin{aligned}-\tau_2\int_{t-\tau_2}^{t}\dot{x}^T(s)R_3\dot{x}(s)ds \le\\ -(x(t)-x(t-\tau(t))R_3(x(t)-x(t-\tau(t)))^T\\ -(x(t-\tau(t)-x(t-\tau_2))R_3(x(t-\tau(t)-x(t-\tau_2))^T\\ -\alpha_2(x(t)-x(t-\tau(t))R_3(x(t)-x(t-\tau(t)))^T\\ -\beta_2(x(t-\tau(t))-x(t-\tau_2))R_3(x(t-\tau(t))-x(t-\tau_2))^T\end{aligned} \quad (29)$$

$$\text{Where}\ \ \alpha_2 = \frac{\tau_2-\tau(t)}{\tau_2} and\ \beta_2 = \frac{\tau(t)}{\tau_2}$$

Therefore, from (28), (29), we can major (26) as

$$\dot{V}_2(t) \le \zeta^T(t)\Xi^q\zeta(t),\ q=1,2 \quad (30)$$

With Ξ^q given in (17) and(18) respectively. Now, let us focus on the third term of (19), we have

$$\dot{V}_3(t) = (\tau_2-\tau_1)\dot{x}^T(s)S\dot{x}(s) - \int_{t-\tau_2}^{t-\tau_2}\dot{x}^T(s)S\dot{x}(s)ds \quad (31)$$

Let us come back to the whole derivative of the LKFs (19).From (24) - (31), the inequality (23) is satisfied if

$$\zeta^T(t)\left(\sum_{i=1}^{2}\Phi_i + \Xi^q\right)\zeta(t) - \int_{t-\tau_2}^{t-\tau_1}\dot{x}^T(s)S\dot{x}(s)ds < 0 \quad (32)$$

With Φ_2 given in (15).

Note that, from the well-known additive property of integration on intervals, we can write:

$$-\int_{t-\tau_2}^{t-\tau_1}\dot{x}(s)S\dot{x}(s)ds \le -\int_{t-\tau_2}^{t-\tau(t)}\dot{x}(s)S\dot{x}(s)ds - \int_{t-\tau(t)}^{t-\tau_1}\dot{x}(s)S\dot{x}(s)ds \quad (33)$$

As well as introduce the following null terms:

$$x(t-\tau(t)) - x(t-\tau_2) - \int_{t-\tau_2}^{t-\tau(t)}\dot{x}(s)ds = 0 \quad (34)$$

$$x(t-\tau_1) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t-\tau_1}\dot{x}(s)ds = 0 \quad (35)$$

Note that the following inequalities hold

$$\begin{aligned}-2\zeta(t)^T N^1\int_{t-\tau(t)}^{t-\tau_1}\dot{x}(s)ds\\ \le(\tau(t)-\tau_1)\zeta(t)^T N^1 S^{-1} N^{1T}\zeta(t)\\ +\int_{t-\tau(t)}^{t-\tau_1}\dot{x}^T(s)S\dot{x}(s)ds\end{aligned} \quad (36)$$

$$\begin{aligned}-2\zeta(t)^T N^2\int_{t-\tau_2}^{t-\tau(t)}\dot{x}(s)ds\\ \le(\tau_2-\tau(t))\zeta(t)^T N^2 S^{-1} N^{2T}\zeta(t)\\ +\int_{t-\tau_2}^{t-\tau(t)}\dot{x}^T(s)S\dot{x}(s)ds\end{aligned} \quad (37)$$

Thus, from (36) and (37) the inequality (32), is satisfied if:

$$\zeta^T(t)\left(\sum_{i=1}^{3}\Phi_i + \Xi^q + (\tau(t)-\tau_1)N^1S^{-1}N^{1T} + (\tau_2-\tau(t))N^2S^{-1}N^{2T}\right)\zeta(t) < 0 \quad (38)$$

With Φ_3 given in (16).Then, with G defined in (13), let us rewrite the NCS model(6) with network-induce delay (3) as:

$$G\zeta(t) = 0 \quad (39)$$

The lemma 2 can be applied if there exists T in ($\mathcal{R}^{5n\times n}$) such that

$$\zeta^T(t)H_e(TG)\zeta(t) = 0 \quad (40)$$

Now, let consider the whole derivative of the LKF (23)-(40), the inequality (23) is satisfied if:

$$\zeta^T(t)\left(\sum_{i=0}^{3}\Phi_i + \Xi^q + H_e(TG) + (\tau(t)-\tau_1)N^1S^{-1}N^{1T} + (\tau_2-\tau_1)N^2S^{-1}N^{2T}\right)\zeta(t) < 0 \quad (41)$$

Applying lemma 3, it yields for q = 1,2:

$$\zeta^T(t)\left(\sum_{i=0}^{3}\Phi_i+\Xi^q+H_e(TG)+(\tau_2-\tau_1)N^qS^{-1}N^{qT}\right)\zeta(t)<0 \qquad (42)$$

Now, applying Schur complement the LMI (42) is equivalent to (12). This completes the proof.

3.2.Controller design for NCS

In this section, when the feedback gain K is unknown, theconditions expressed in theorem1are no more LMIs. The following theorem provides a convexification procedure for the design of networked controller (4) such that the closed loop NCS (6) is stabilized.

Theorem.2. Let $i\in I_2$ and $j\in I_3$ for given scalars $\tau_i>0$ such that $\tau_1\le\tau(t)\le\tau_2$, the NCS model (6) is asymptotically stabilized by the NCS controller (4) If there exist symmetric positive definite matrices $X,\bar{P},\bar{S},\bar{Q}_i$ and $\bar{R}_j(in\ \mathcal{R}^{n\times n})$ and matrices variables $\bar{N}^1$ and $\bar{N}^2(in\ \mathcal{R}^{5n\times n})$, and two arbitraryscalars ε_1 and ε_2 such that the following LMIsconditionshold forboth q = 1 and q = 2:

$$\begin{bmatrix}\sum_{i=1}^{3}\bar{\Phi}_i+\bar{\Xi}^q+\tilde{X} & (\tau_2-\tau_1)\bar{N}^q\\ * & -(\tau_2-\tau_1)\bar{S}\end{bmatrix}<0 \qquad (43)$$

Where:

$$\tilde{X}=\begin{bmatrix}AX & 0 & BF & 0 & -X\\ 0&0&0&0&0\\ \varepsilon_1AX & 0 & \varepsilon_1BF & 0 & -\varepsilon_1X\\ 0&0&0&0&0\\ \varepsilon_2AX & 0 & \varepsilon_2BF & 0 & -\varepsilon_2X\end{bmatrix},$$

$$\bar{\Phi}_1=H_e(e_1^T\bar{P}e_5)+e_1(\bar{Q}_1+\bar{Q}_2)e_1^T-e_2\bar{Q}_1e_2^T-e_4\bar{Q}_2e_4^T, \quad (44)$$

$$\bar{\Phi}_2=e_5(\tau_2-\tau_1)\bar{S}_1e_5^T, \quad (45)$$

$$\bar{\Phi}_3=H_e\left(\bar{N}^1\left(e_3^T-e_4^T\right)+\bar{N}^2\left(e_2^T-e_3^T\right)\right) \quad (46)$$

$$\begin{aligned}\bar{\Xi}^1=&\,e_5\left(\tau_1^2\bar{R}_1+\tau_2^2\bar{R}_2+(\tau_2-\tau_1)^2\bar{R}_3\right)e_5^T-(e_1-e_2)\bar{R}_1(e_1-e_2)^T\\&-(e_2-e_3)\bar{R}_2(e_2-e_3)^T-(e_3-e_4)\bar{R}_2(e_3-e_4)^T\\&-(e_3-e_4)\bar{R}_2(e_3-e_4)^T-\frac{\tau_1}{\tau_2}(e_1-e_3)\bar{R}_3(e_1-e_3)^T\\&-(e_1-e_3)\bar{R}_3(e_1-e_3)^T-(e_3-e_4)\bar{R}_3(e_3-e_4)^T\\&-\frac{\tau_2-\tau_1}{\tau_2}(e_3-e_4)\bar{R}_3(e_3-e_4)^T, \qquad (47)\end{aligned}$$

$$\begin{aligned}\bar{\Xi}^2=&\,e_5\left(\tau_1^2\bar{R}_1+\tau_2^2\bar{R}_2+(\tau_2-\tau_1)^2\bar{R}_3\right)e_5^T-(e_1-e_2)\bar{R}_1(e_1-e_2)^T\\&-(e_2-e_3)\bar{R}_2(e_2-e_3)^T-(e_3-e_4)\bar{R}_2(e_3-e_4)^T\\&-(e_1-e_3)\bar{R}_3(e_1-e_3)^T-(e_3-e_4)\bar{R}_3(e_3-e_4)^T\\&-(e_2-e_3)\bar{R}_2(e_2-e_3)^T-(e_1-e_3)\bar{R}_3(e_1-e_3)^T \qquad (48)\end{aligned}$$

Proof2: Let $T=\left[X^{-T}\ \ 0\ \ \varepsilon_1X^{-T}\ \ 0\ \ \varepsilon_1X^{-T}\right]^T$ and $D_X=diag\left[X\ \ X\ \ X\ \ X\ \ X\right]^T$. Pre and post-multiplying (12) by D_X and its transpose respectively and with the change of variables $F=KX,\bar{P}=X^TPX,\bar{S}=X^TSX,\bar{Q}_2=X^TQ_2X$ $\bar{R}_i=X^TR_iX, i\in\mathbf{I}_3,\bar{Q}_l=X^TQ_lX$, and $\bar{N}_l^q=X^TN_lX(l,q\in I_2)$.

Oneobtains the conditions expressed intheorem2. This completes the proof.

4. Numerical examples

In order to illustrate the effectiveness of the main results obtained in the previous section, we provide a comparison with those of some well-known numerical examples.

Example1: Consider the following NCS model:

$$A=\begin{bmatrix}0&1\\-1&-2\end{bmatrix}, B\times K=\begin{bmatrix}0&0\\-1&-1\end{bmatrix}.$$

For various given value of τ_1, the allowable upper bounds τ_2 which guarantee the asymptotic Stability of system (6) are listed in Table (1).

Among all the considered results it can be seen that the stability results obtained in thispaper are less conservative.

Table 1: maub(τ_2)with varying τ_1 of Example 1.

Considered results	2	3	4	5
[12]	2.4884	3.3403	4.3424	5.2970
[15]	2.5608	3.4542	4.3787	5.3228
[7]	2.6134	3.50464	4.4271	5.3696
[5]	2.6344	3.5124	4.4304	5.3709
Theorem 1	**2.6684**	**3.5688**	**4.4867**	**5.4230**

To show that our approach is more effective to reduce conservatism, a simulation of the NCS has been realized with the initial condition are:

$x(0)=[-0.5 \quad 0.5]^T$

and $2 \le \tau(t) \le 2.6684$.

The state trajectories of the system (6) with parameters described in example 1 and network-induced delay (3) are plotted in Fig.1, which shows that the NCS (6) is asymptotically stable.

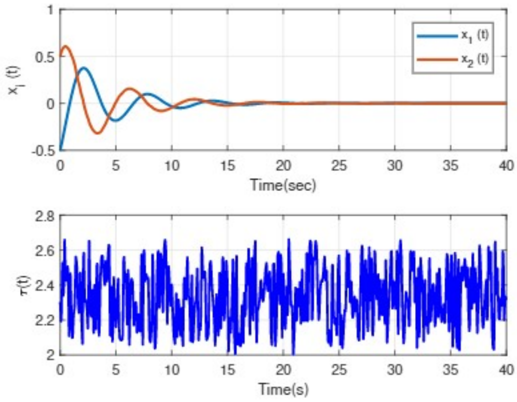

Figure 1: State response and network-induced delay signals of Example 1.

Example 2: Network control of a satellite model.In this example, the satellite model investigated in [6] is considered, this system "without disturbance" can be represented by the following state space matrices :

$$A=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.3 & 0.3 & -0.004 & 0.004 \\ 0.3 & -0.3 & 0.004 & -0.004 \end{bmatrix}, B=\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

The goal is to design the gain matrix K such that networked controller is asymptotically stabilizing the satellite model (4). By using theorem 2 with $\tau_1=0.01, \varepsilon_1=0.5$ and $\varepsilon_2=0.3$ the maximum allowable upper bound of network-induced delay $maub(\tau_2)=0.6$ is under the state feedback gain $K=[-0.1622 \quad 0.0422 \quad -0.6699 \quad -0.1254]$.

For this case, Fig.2 shows the NCS closed-loop trajectories of the satellite and the network-induced delay as well as the control signal from the initial state $x(0)=[-0.1 \quad 0.5 \quad -0.3 \quad 0.2]^T$,

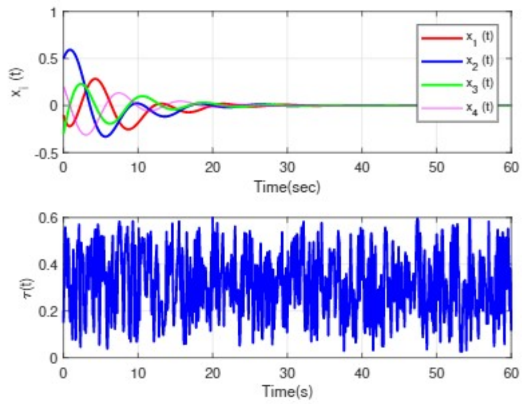

Figure 2: States response, network-induced delay signals of satellite model.

We can notice that the satellite model is properly stabilized by the designed NCS controller.

Conclusion

In this paper, a new stability criteria using Lyapunov-Krasovskii for sampled-data NCSs model with time-varyingdelays is presented. The resulting LKF's cross terms are estimated via new integral inequalities which allowing more stability conservativeness compared with other methods.
Arobust state feedback controller design approach is also derived from the new stability criteria.

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