Fixed-time Synergetic Control for Synchronization of Fractional-order Chaotic Systems

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Abstract: In this paper, a fixed-time synergetic control is proposed for the synchronization of fractional-order chaotic systems. It is similar to the sliding mode approach but without chattering and achieves exact convergence of the macro-variable. Synergetic control is a robust approach that does not require linearization and is suitable for real-time implementation since it does not require a discontinuous term in its control law. The Lyapunov framework is used to ensure the convergence of the controlled system. Computer simulations are performed to illustrate the effectiveness of the proposed method.

Keywords: Fractional-order Chaotic Systems, Synergetic Control, fixed-time, Chaotic synchronization.

1. INTRODUCTION

Fractional calculus is an old mathematical with a 300 years old history. For many years, this branch of science was considered as a sole mathematical and theoretical subject with nearly no applications [1]. But, recently, fractional calculus has received great attention in various fields of scientific research. The main features of fractional-order calculus is that it has an infinite memory, high degrees of freedom and more flexibility, so we consider it as the best method for describing more precisely many physical phenomena [2, 3]. Several works have used the fractional-order differential operators to construct fractional-order observers and to design robust fractional-order controllers [4-6].

The majority of physical systems exhibit nonlinear and complex dynamical behavior. One of the most intriguing complex nonlinear phenomena is the irregular behavior of deterministic systems, which has implications in several areas and is called chaos [7].

E.N. Lorenz of the Department of Meteorology at MIT discovered chaotic phenomena in 1963 when he published a system of differential equations describing a simplified atmospheric model with non-periodic steady-state behavior that expresses the essence of turbulence [8].

Chaos exists in chemical reactions, lasers, electronic circuits, fluid dynamics, etc. Moreover, the existence of chaos is confirmed in natural systems such as weather, solar system, heart and brain of living creatures [8]. Their particularities, besides instability, are a high sensitivity to initial conditions, system parameters and a seemingly random evolution. Therefore, these characteristics have attracted many researchers to tackle the challenging problems of chaos control and synchronization of chaotic systems [9].

With the development of the fractional calculus, special attention has been given to the study of the fractional-order behavior of chaotic systems. The basic idea is to replace the integer-order derivative with a fractional-order derivative in some well-known nonlinear integer chaotic systems [10].

The synchronization of fractional-order chaotic systems has attracted interest in various fields such as cryptography, secure communications, pattern recognition, and nonlinear control systems. Several approaches to the synchronization of fractional-order chaotic systems have been reported in the literature, including the methods based on nonlinear observers, we can cite for example [10, 11].

On the other hand, synchronization can be considered as a control problem, many types of control methods have been used, including feedback control [12], backstepping control [13], sliding mode control (SMC) [14]. However, in practical applications of SMC, the designer may experience undesirable oscillations with finite frequency and amplitude, which is known as chattering phenomenon [15]. Chattering is a ruinous phenomenon because it reduces control accuracy, excites fast dynamics that were

neglected in the ideal model, induces instability, and may cause severe damage and high wear of moving mechanical parts of actuators due to high frequency control effort [16].

Several solutions have been proposed to overcome this problem, the method that has similarities with the sliding mode technique is the approach based on the synergetic theory. The synergetic theory was introduced by Kolesnikov, the Russian scientist proposed the essential rules of nonlinear system synthesis theory based on the synergetic realization in addition to its applications, which is known as the synergetic control theory [17].

The controller designed by synergetic theory is attractive because of its simplicity and its optimal feature, it eliminates chattering as a whole by the use of a completely continuous control law [18].

In this paper, fractional-order synergetic control theory has been investigated to design a fixed-time synergetic controller dedicated to the synchronization of two identical fractional-order chaotic systems with different initial conditions and two different fractional-order chaotic systems.

The rest of the paper is organized as follows. Section 2 contains some definitions of fractional order calculus. A brief overview of synchronization of fractional chaotic systems is presented in Section 3, followed by fixed-time stability theory. In section 5, the fractional synergetic controller is designed, then it's applied to the synchronization of fractional-order chaotic systems and the simulation results are discussed. Finally, the paper ends with a conclusion.

2. PRELIMINARIES

A. Fractional calculus and fractional-order systems.

In this subsection, some definitions and properties with respect to fractional-order integral and derivatives are recalled.

Definition 1: [19] The Riemann–Liouville fractional integral of order $\alpha, 0 < \alpha < 1$, of a real-valued function $x(t)$ with respect to the variable $t \in \mathbb{R}_+$which represents the time, is defined as

$$I_t^{\alpha} x(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} \frac{1}{(t-\tau)} x(\tau) d\tau \qquad (1)$$

Where t_0 denotes the initial time and Euler's Gamma function $\Gamma(\alpha)$ is defined as

$$\Gamma(\alpha) = \int_0^{\infty} v^{\alpha-1} e^{-v} dv \qquad (2)$$

Definition 2: [19] The Riemann–Liouville derivative of a given real order$\alpha \in \mathbb{R}_+^*, 0 < \alpha < 1$, on a real-valued function $x(t)$ with respect to the variable $t \in \mathbb{R}_+$is defined as

$${}_{t_0}^{RL}D_f^{\alpha} x(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \left\{ \int_{t_0}^{t} \frac{1}{(t-\tau)} x(\tau) d\tau \right\} \qquad (3)$$

Some useful properties for establishing the results proposed in this paper are given below [19].

$$\begin{gathered} D^{\alpha} \lambda x(t) = \lambda D^{\alpha} x(t) \quad \forall \lambda \in \mathbb{R} \\ D^{\alpha}\left(I^{\alpha} x(t)\right) = x(t) \\ D^{\alpha 1}\left(I^{\alpha 2} x(t)\right) = D^{\alpha 1 - \alpha 2} x(t) \quad \forall \alpha_1, \alpha_2 \in \mathbb{R}_+, \alpha_1 > \alpha_2 \\ D^{\alpha 1}\left(I^{\alpha 2} x(t)\right) = I^{\alpha 2 - \alpha 1} x(t) \quad \forall \alpha_1, \alpha_2 \in \mathbb{R}_+, \alpha_2 > \alpha_1 \end{gathered} \qquad (4)$$

3. SYNCHRONISATION OF FRACTIONAL-ORDER CHAOTIC SYSTEMS

The synchronisation of two fractional-order chaotic systems is a challenging task which has been tackled trough many robust approaches. In this paper a fixed-time synergetic control is used to force a slave chaotic system output to follow the output of a master chaotic system.
The master fractional-order chaotic system is given by

$$\begin{aligned} D^{\alpha} x_i(t) &= x_{i+1}(t) \qquad i = 1 \ldots n-1 \\ D^{\alpha} x_n(t) &= f(x,t) \end{aligned} \qquad (5)$$

The slavefractional-order chaotic system is given by

$$\begin{aligned} D^{\alpha} z_i(t) &= z_{i+1}(t) \qquad i = 1 \ldots n-1 \\ D^{\alpha} z_n(t) &= f(z,t) + g(z,t)u(t) \end{aligned} \qquad (6)$$

Where $f(x,t)$ and $f(z,t)$ represent fractional-orderchaotic systems dynamics and $g(z,t)$ is the control gain function, and $u(t)$ is the synchronizing control, and $\alpha$is the fractional-order derivative.

Let e_i(t) = z_i(t)- x_i(t), $i = 1 \ldots n$ be the synchronization errors between master and slave, their dynamics are given as

$$\begin{aligned} D^{\alpha} e_i(t) &= e_{i+1}(t) \qquad i = 1 \ldots n-1 \\ D^{\alpha} e_n(t) &= f(e,t) + g(z,t)u \end{aligned} \qquad (7)$$

The synchronization consists in elaborating a control law $u(t)$such that e_i, $i = 1, \ldots, n-1$converge to zero despite the

different initial conditions or dynamical systems.

4. FIXED-TIME STABILITY

The control objective is to design a fixed-time fractional-order synergetic algorithm for system dynamics stabilization. The following required definitions are given:

Definition 3.[20] Consider the following differential equation system with $x \in \mathbb{R}$ and the nonlinear function $f(x) \in \mathbb{R}$.

$$\dot{x}(t) = f(x) \quad , x(0) = x_0 \quad (8)$$

Assume the origin is an equilibrium point of System (8), then it is called a fixed-time stable provided that it is stable with bounded convergence time $T(x_0)$, that is $\exists T_{max} > 0$, such that $\lim_{X_0 \to \infty} [T(x_0)] \leq T_{max}$.

Lemma 1. [20, 21]Consider the following differential equation system with $y \in \mathbb{R}$

$$\dot{y}(t) = -\delta y^{\frac{m}{n}} - \beta y^{\frac{q}{p}} \quad , y(0) = y_0 \ (9)$$

Where δ and β are> 0, and all the parameters $\mathrm{m}, \mathrm{n}, \mathrm{q}$ and p are odd and positive numbers, satisfying $\frac{m}{n} > 1, 0 < \frac{q}{p} < 1$. The convergence time of(9)for stabilizing to the origin is set to be $T(y_0)$, then y will converge to the origin within an upper bounded constant fixed-time T_{max}(y), that $\lim_{y \to \infty} [T(x_0)] \leq T_{max}(y)$.

$$T_{max}(y) = \frac{1}{\delta} \frac{n}{m-n} + \frac{1}{\beta} \frac{p}{p-q} (10)$$

Lemma 2. [21]Consider the following differential equation system with $V$is a positive definite function:

$$\dot{V} = \delta V^{\varsigma 1} - \beta V^{\varsigma_2}, \quad V(0) = V_0 (11)$$

Where δ and $\beta$are positive real numbers, ς_1 and ς_2 are positive numbers that satisfy$\varsigma_1 > 1, 0 < \varsigma_2 < 1$. The convergence time of (11) to stabilize to the origin is set to be $T(V_0)$, then V will converge to the origin within an upper bounded constant fixed-time $T_{max}(V)$, that is $\lim_{V_0 \to \infty} [T(V_0)] \leq T_{max}(V)$ and

$$T_{max}(V) = \frac{1}{\delta} \frac{1}{\varsigma_1 - 1} + \frac{1}{\beta} \frac{1}{1-\varsigma_2} \quad (12)$$

5. SYNERGETIC CONTROL DESIGN

Consider the following fractional-order nonlinear system

$$D^{\alpha} x_i(t) = x_{i+1}(t) \quad i = 1, \dots, n-1$$
$$D^{\alpha} x_n(t) = f \ (x, t) + g_n(x, t) u(t) (13)$$

where $x \in \mathbb{R}^n$ is the state variable vector of the system, $f \ (x, t) \in \mathbb{R}$ represents a smooth nonlinear function describing the system dynamics, $g_n(x, t) \neq 0$ is the control gain function, and $u \in \mathbb{R}$ is the input control.

The control vector u to be found based on synergetic control theory guarantees system dynamics movement from any initial state to invariant manifold and then toward the origin. The control designed is a function of macro-variable ψ. These macro-variables ψ are defined as a function of state variables or errors of the dynamical system and should be chosen properly by designer and satisfy

$$T\dot{\psi} + \phi(\psi) = 0 \quad (14)$$

where T is a design parameter that specifies the convergence rate of the macro-variable ψ to the invariant manifold $\psi(x, \mathrm{t}) = 0$, and $\phi(\psi$) is a smooth differentiable function of ψ that is chosen such that [22].

$$\phi(0) = 0$$
$$\phi(\psi)\psi > 0, \ \forall \psi \neq 0 (15)$$

Lemma 3. [22]If the function $\phi(\psi)$ is selected in the form of (16) then $\phi(\psi)$is satisfying previous conditions (15)

$$\phi(\psi) = \psi^{\frac{p}{q}} + \psi^{\frac{q}{p}} \quad (16)$$

where the parameters q and p are odd number such that $q > 0$ and $p > 0$, and 0 $<\frac{q}{p}<1$.

Then according to Lemma 3 the aggregated macro-variable dynamics can be written as follows

$$T\dot{\psi} + \psi^{\frac{p}{q}} + \psi^{\frac{q}{p}} = 0 \quad (17)$$

Therefore, according to Lemma 1 and Equation (17) the macro-variable ψ converges in a fixed-time toward the invariant manifold $\psi(x, \mathrm{t}) = 0$ and remains on it.
The convergence time, according to Lemma 1, is given by $T(\psi_0)$ and bounded by a constant $T_{max}(\psi)$ such that:

$$T_{max}(\psi) = T \frac{p+q}{p-q} \ (18)$$

where p, q and T are all design parameters selected such that the macro-variable attracted to the invariant manifold as fast as required.
In order to design the controller for system (13), the macro-variable ψ is given as

$$\psi = k_1 I^{(1-\alpha)} x_1(t) + k_2 I^{(1-\alpha)} x_2(t) + \cdots + I^{(1-\alpha)} x_n(t) (19)$$

where k_i for $i = 1,2,\dots,n-1$ are the convergence parameters.
By taking the time derivative of the macro-variable (19) and using the properties of (4) we obtain

$$\dot{\psi} = k_1 D^{\alpha} x_1(t) + k_2 D^{\alpha} x_2(t) + \cdots + k_n D^{\alpha} x_n(t)$$

Then

$$\dot{\psi} = k_1 D^{\alpha} x_1(t) + k_2 D^{\alpha} x_2(t) \dots + k_n (f\ (x,t) + g_n(x,t)u(t)) \quad (20)$$

By substituting (20) into (17)

$$T(k_1 D^{\alpha} x_1(t) + k_2 D^{\alpha} x_2(t) + \cdots + k_n (f\ (x,t) + g_n(x,t)u(t))) + \psi^{\frac{p}{q}} + \psi^{\frac{q}{p}} = 0 \quad (21)$$

The control input $u(t)$, it can be written in the following form

$$u(t) = -\frac{1}{g_n(x,t)}[k_1 D^{\alpha} x_1(t) + k_2 D^{\alpha} x_2(t) + \cdots + k_n f\ (x,t) + \frac{\psi^{\frac{p}{q}}}{T} + \frac{\psi^{\frac{q}{p}}}{T}] \quad (22)$$

Theorem 1. Consider the System (13) with the control input given by (22), and assume that the selected macro-variable (19) reaches the invariant manifold $\psi(x,\mathrm{t}) = 0$ within a fixed-time. Then, the state variables x_i, $i = 1,2,\dots,n$ of System (13) converge to desired state asymptotically.

Proof. Consider the following Lyapunov candidate function manifold $V(\mathrm{t})$that is defined as follow:

$$V(\mathrm{t}) = \frac{1}{2}\psi^2(t) \quad (23)$$

Then the time derivative of Lyapunov function $V(t)$, can be obtained as follows:

$$\dot{V}(\mathrm{t}) = \psi(t)\dot{\psi}(t) \quad (24)$$

Substitute (20) into (24), and then

$$\dot{V}(\mathrm{t}) = \psi(t)\big(k_1 D^{\alpha} x_1(t) + k_2 D^{\alpha} x_2(t) + \cdots + k_n D^{\alpha} x_n(t)\big) \quad (25)$$

then

$$\dot{V}(\mathrm{t}) = \psi(t)(k_1 D^{\alpha} x_1(t) + k_2 D^{\alpha} x_2(t) + \cdots + k_n (f(x,t) g_n(x,t) u(t))) \quad (26)$$

$$= \psi(t)\Bigg(k_1 D^{\alpha} x_1(t) + k_2 D^{\alpha} x_2(t) + \cdots + k_n (f(x,t) + g_n(x,t))\Bigg(-\frac{1}{g_n(x,t)}\Bigg[k_1 D^{\alpha} x_1(t) + k_2 D^{\alpha} x_2(t) + \cdots + f\ (x,t) + \frac{\psi^{\frac{p}{q}}}{T} + \frac{\psi^{\frac{q}{p}}}{T}\Bigg]\Bigg)\Bigg) \quad (27)$$

$$= \psi(t)\Bigg(k_1 D^{\alpha} x_1(t) + k_2 D^{\alpha} x_2(t) + \cdots + f(x,t) - k_1 D^{\alpha} x_1(t) - k_2 D^{\alpha} x_2(t) - \cdots - k_n f\ (x,t) - \frac{\psi^{\frac{p}{q}}}{T} - \frac{\psi^{\frac{q}{p}}}{T}\Bigg) \quad (28)$$

$$= -\frac{1}{T}\psi\left(\psi^{\frac{p}{q}} + \psi^{\frac{q}{p}}\right)$$

$$= -\frac{1}{T}\psi\left(\psi^{\frac{p}{q}+1} + \psi^{\frac{q}{p}+1}\right)$$

$$= -\frac{1}{T}\left(2V^{\left(\frac{p+q}{\frac{q}{2}}\right)} + 2V^{\left(\frac{p+q}{\frac{p}{2}}\right)}\right) \quad (29)$$

Let $W = 2V$

$$\dot{W}(t) = -\frac{2}{T}\left(W^{\left(\frac{p+q}{2q}\right)} + W^{\left(\frac{p+q}{2p}\right)}\right)$$

$$= -\frac{2}{T}(W^{\gamma_1} + W^{\gamma_2}) \quad (30)$$

With $\gamma_1 = \left(\frac{p+q}{2q}\right)$ and $\gamma_2 = \left(\frac{p+q}{2p}\right)$

According to Lemma 2, and since $\gamma_1 > 1$ and $0 < \gamma_2 < 1$ then the function W and V converge within fixed-time to zero, which forces the defined macro-variable ψ to reach the invariant manifold $\psi(x,t) = 0$ within a fixed-time. Moreover, on this manifold the state variables asymptotically converge to the desired state.
This completes the proof.

6. APPLICATION TO FRACTIONAL-ORDER CHAOTIC SYSTEMS SYNCHRONIZATION

This section is devoted to illustrating the efficiency of the proposed fixed-time synergetic control applied to the synchronization of fractional-order chaotic systems.

The simulations of the chaotic systems are carried out by means of the Grunwald-Letnikov formula of the derivative of the fractional-order, given by [19]:

$$\Delta^{\alpha} f(t) = \frac{1}{h^{\alpha}} \sum_{j=0}^{j=\left(\frac{t}{h}\right)} (-1)^j \binom{\alpha}{j} f(t - jh)$$

$$\binom{\alpha}{j} = \frac{\alpha(\alpha-1)(\alpha-2)\dots(\alpha-j+1)}{j!}$$

A. Example 1:

Consider the master fractional-order Genesio-Tesi system [23]:

$$D^{\alpha}x_1(t) = x_2(t)$$
$$D^{\alpha}x_2(t) = x_3(t) \quad (31)$$
$$D^{\alpha}x_3(t) = -x_1(t) - 1.1\,x_2(t) - 0.15x_3(t) + x_1^2(t)$$

With the following initial conditions

$x_{10} = I_a^{(1-\alpha)}x_1(t)|_{t=0} = 0.5,$

$x_{20} = I_a^{(1-\alpha)}x_2(t)|_{t=0} = -0.5,$

$x_{30} = I_a^{(1-\alpha)}x_3(t)|_{t=0} = 0.5.$

Consider the slavefractional-order Genesio-Tesi system [22]:

$$D^{\alpha}z_1(t) = z_2(t)$$
$$D^{\alpha}z_2(t) = z_3(t) \quad (32)$$
$$D^{\alpha}z_3(t) = -z_1(t) - 1.1\,z_2(t) - 0.15z_3(t) + z_1^2(t) + u(t)$$

With the following initial conditions

$z_{10} = I_a^{(1-\alpha)}z_1(t)|_{t=0} = -0.5,$

$z_{20} = I_a^{(1-\alpha)}z_2(t)|_{t=0} = 0.5,$

$z_{30} = I_a^{(1-\alpha)}z_3(t)|_{t=0} = -0.5.$

The selected macro-variable is given as

$$\psi = k_1 I^{(1-\alpha)}e_1 + k_2 I^{(1-\alpha)}e_2 + k_3 I^{(1-\alpha)}e_3 \quad (33)$$

The simulation results have been carried using the following parameters values: $T = 0.1$, $p = 9$, $q = 7$, $k_1 = k_2 = k_3 = 1$ and $\alpha = 0.9$ is the commensurate fractional-order derivative.

$$u(t) = -k_1 D^{\alpha}e_1(t) - k_2 D^{\alpha}e_2(t) - k_3 f(e,t) - \frac{\psi^{\frac{p}{q}}}{T} - \frac{\psi^{\frac{q}{p}}}{T} \quad (34)$$

According to Theorem 1, with the selected macro-variable (33) reaches into the invariant manifold within a fixed-time and with the control input (34), the errors $D^{\alpha}e_i(t), i = 1,2,3.$ asymptotically converge to zero. Synchronization between the fractional-order slave chaotic system (32) and the fractional-order master chaotic system (31) is then achieved.

Figures 1 and 2 show the time response of the errors with and without control. Fig. 3 shows the time response of the macro-variable. As shown in these figures, synchronization is successfully achieved. The settling time is given by the upper bound $T_{max}(\psi) = 0.8s$. The control input is shown in Fig. 4.

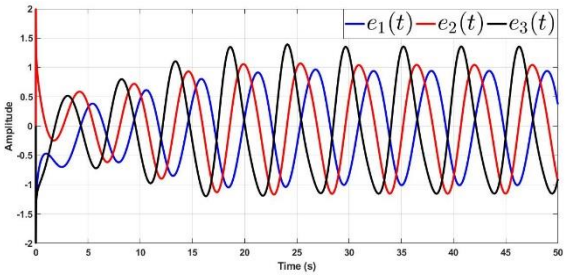

Fig.1 Synchronization errors without control.

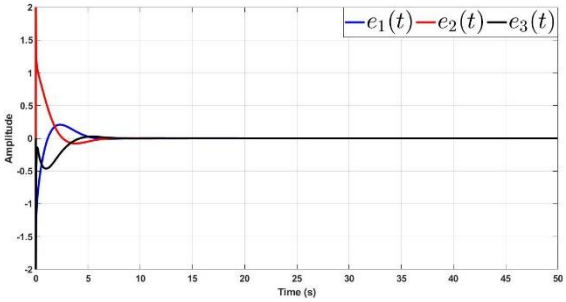

Fig. 2 Synchronization errors with control.

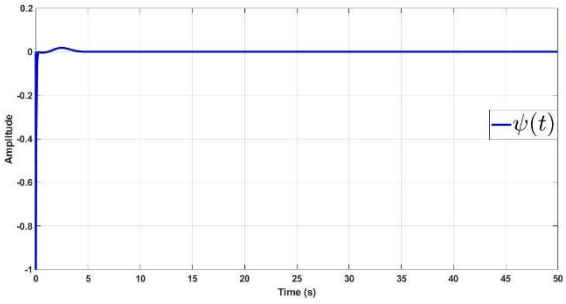

Fig. 3 Macro-variable.

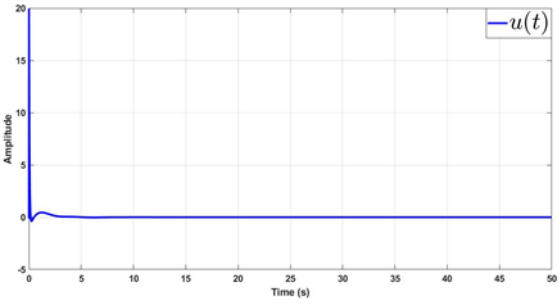

Fig. 4. Input control.

B. Example 2:

In this example, we will consider the same master system as in the first example, but for the slave system we will consider the fractional-order Arneodo system [23]:

$$D^{\alpha}z_1(t) = z_2(t)$$

$$D^{\alpha}z_2(t) = z_3(t)(35)D^{\alpha}z_3(t) = 5.5z_1(t) - 3.5\,z_2(t) - z_3(t) - z_1^3(t) + u(t)$$

With the following initial conditions

$z_{10} = I_a^{(1-\alpha)} z_1(t)|_{t=0} = -0.25,$

$z_{20} = I_a^{(1-\alpha)} z_2(t)|_{t=0} = 0.25,$

$z_{30} = I_a^{(1-\alpha)} z_3(t)|_{t=0} = -0.25.$

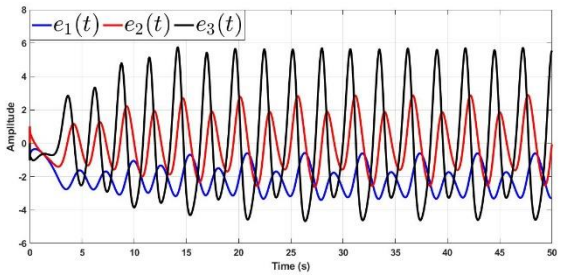

Fig. 5 Synchronization errors without control.

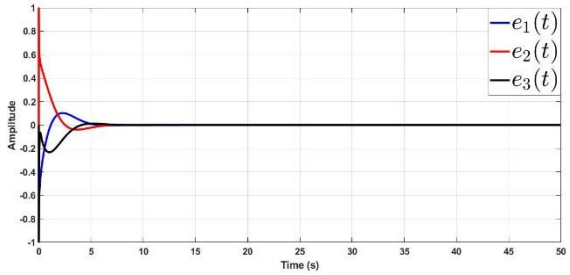

Fig. 6 Synchronization errors with control.

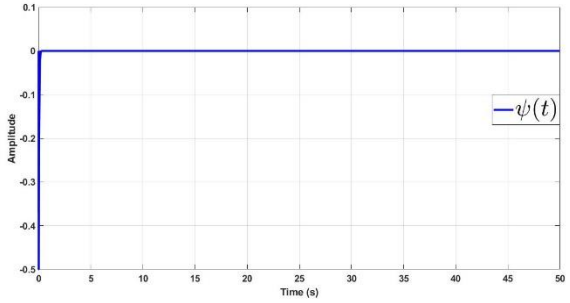

Fig. 7 Macro-variable.

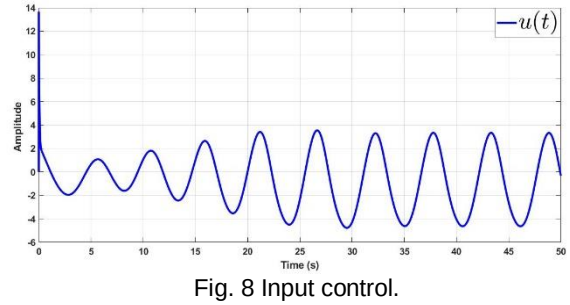

Fig. 8 Input control.

Figures 5 and 6 display the time response of the errors with and without control, while Fig. 7 presents the time response of the macro variable. These figures demonstrate that successful synchronization has been achieved. The upper bound settling time $T_{max}(\psi) = 0.8s$. The control input is depicted in Fig. 8.

Then the synchronization between the fractional-order slave chaotic system (35) and the fractional-order master chaotic system (31) is achieved.

The simulation results show that the effectiveness of the proposed approach is well verified and the synchronization is achieved. The main advantages of the proposed method are:

- The proposed fractional-order synergetic control does not introduce a chattering phenomenon.
- The convergence of the macro-variable is guaranteed in fixed-time and its convergence time is uniformly bounded by a predefined value.

7. CONCLUSION

This paper focuses on the fixed-time synergetic control for the synchronization of fractional-order chaotic systems. In particular, this paper combines the fractional-order synergetic control and the fixed-time control method. Furthermore, the synchronization has been achieved despite the different initial conditions and different chaotic systems. In addition, the proposed control algorithm, other than being without chattering and achieve the convergence of the macro-variable in fixed-time.

The Lyapunov framework is exploited to ensure the convergence of the controlled system. Finally, the computer simulation is given to show the feasibility of the main results.

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