Design of an Optimal Multivariable PID Controller for a Two-Link Robot Arm

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Abstract: Incorporating manipulator robots into various industries has revolutionized automation and manufacturing processes. Manipulator robots are versatile machines equipped with robotic arms and end-effectors, enabling them to handle complex tasks with precision and efficiency. This research aims to design an optimal Mutivariable PID (MPID) controller that ensures precise and accurate movements of two-link robot arm. Theoretical foundations of the MPID controller are discussed together with its relevance to robotic arm systems, and the challenges associated with its implementation. The research methodology involves mathematical modelling of the two-link robot arm dynamics, and the use of an optimization algorithm to determine the optimal MPID controller coefficients. Simulation results demonstrate performance enhancement and productivity of the two-link robot arm through the improved MPID controller.

Keywords: PID, MPID, Manipulator robot, optimal PID, controller, Two-link robot arm

1. INTRODUCTION

The control of robotic systems has become an essential field in automatic and systems engineering. Robots are widely employed in various industries, ranging from manufacturing to healthcare, where they perform tasks with precision and efficiency. In particular, the control of robot arms plays a crucial role in achieving accurate and smooth movements [1], [2], [3].

A two-degree robot arm, also known as a two-link robot arm, consists of two connected links that can rotate around their respective joints. Controlling such a system presents a multivariable control challenge, as the movement of one link affects the position and dynamics of the other link.

The objective of this study is to design a Multivariable Proportional-Integral-Derivative (MPID) controller for a two-link robot arm. PID control is a widely used technique in control systems, offering simplicity and effectiveness. By applying MPID control to both joints of the robot arm, we aim to achieve precise and coordinated movements of the arm [4], [5], [6], [7], [8], [9], [10].

The main motivation behind this research is to enhance the performance of two-link robot arms in terms of accuracy, speed, and stability. This improvement will contribute to the advancement of automation processes, leading to increased productivity and quality in various applications.

The findings of this study have the potential to significantly impact the field of robotics, providing valuable insights into the control of multivariable systems. The successful implementation of the MPID controller can lead to advancements in robotic arm control strategies and open up new possibilities for applications in various industries.

This paper is organized as follows. In section 2, the mathematical model of a two-link robot arm is given. Then, in section 3, the proposed MPID controller for two-link robot arms is discussed. Finally, in the fourth section, simulation results are presented and then discussed.

2. MATHEMATICAL MODEL OF A TWO-LINK ROBOT ARM

A schematic representation of a two-link robot arm is given in Fig. 1.

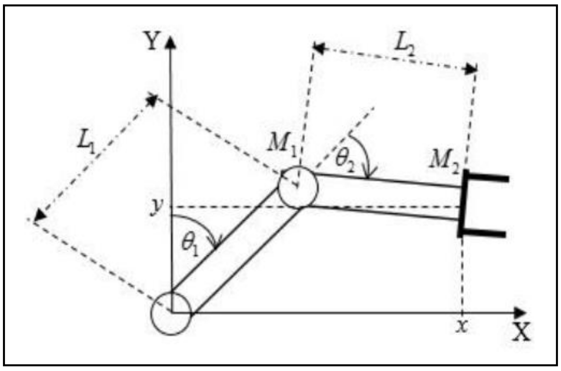


Fig. 1 Two-link robot arm [11].

The dynamic model of a manipulator robot of two links can be written by the following equations:

$$\tau = M(\theta)\ddot{\theta} + C(\theta,\dot{\theta}) + G(\theta) \qquad (1)$$

where

$\theta = [\theta_1 \quad \theta_2]^T$: is the vector of joint variables (output);

$\tau = [\tau_1 \quad \tau_2]^T$: is the vector of applied torques (control input);

$M(\theta)$: is the inertia matrix;

$C(\theta,\dot{\theta})$: is the vector of Coriolis and centrifugal forces;

$G(\theta)$: is a vector of gravity torques;

M, C, and G are given as follows:

$$M(\theta) = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \qquad (2)$$

$$\begin{cases} D_{11} = M_2L_2^2 + (M_1+M_2)L_1^2 + 2M_2L_1L_2cos(\theta_2) \\ D_{12} = D_{21} = M_2L_2^2 + M_2L_1L_2cos(\theta_2) \\ D_{22} = M_2L_2^2 \end{cases} \qquad (3)$$

$$C(\theta,\dot{\theta}) = \begin{bmatrix} -M_2L_1L_2(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2)\sin(\theta_2) \\ +M_2L_1L_2\dot{\theta}_1^2\sin(\theta_2) \end{bmatrix} \qquad (4)$$

$$G(\theta) = \begin{bmatrix} -M_2gL_2\sin(\theta_1+\theta_2) - (M_1+M_2)gL_1\sin(\theta_1) \\ -M_2gL_2\sin(\theta_1+\theta_2) \end{bmatrix} \qquad (5)$$

where

g : The gravitational acceleration;

M1 : the mass of the first link;

M2 : the mass of the second link;

L1 : the length of the first link;

L2 : the length of the second link.

3. DESIGN OF A MULTIVARIABLE PID CONTROLLER

Principle

The synthesis of centralized PI/PID controllers for Multiple-Input Multiple-Output MIMO systems involves designing the gains for each controller in a coordinated manner to achieve the desired performance and stability. Various methods have been proposed in the literature to address this challenge [12], [13]. These methods consider the dynamics of the system, the interactions between variables, and the performance requirements to determine the appropriate gains for each controller. The goal is to minimize the cross-coupling effects and achieve satisfactory control performance for all variables. For that each PID controller is responsible for controlling one specific variable while considering the interactions between variables from joint to another [5], [6], [7].

During the operation, error signals are calculated for each variable by comparing the desired setpoints or trajectories with the actual values. These error signals serve as inputs to the respective MPID controller. Control actions are then calculated using the PID control algorithm for each variable, based on the error signal and the PID gains, the control actions are applied to the corresponding actuators position. The proposed control law is given in (6):

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = MPID(s) \cdot \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \qquad (6)$$

where

$$MPID(s) = \begin{bmatrix} PID_{11}(s) & PID_{12}(s) \\ PID_{21}(s) & PID_{22}(s) \end{bmatrix} \qquad (7)$$

The proposed control structure is illustrated in Fig. 2.

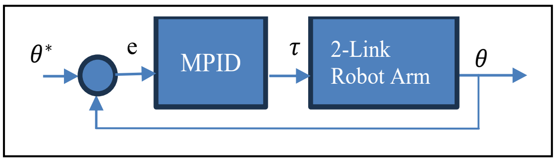


Fig. 2Closed-loop system using MPID.

Tunning

The tuning of the coefficients of the MPID is a crucial step in the development of intelligent controllers. To achieve this, Interior Point methods, a class of optimization techniques, are employed [14], [15].

Interior Point methods are inspired by mathematical optimization and are particularly suited for solving complex and non-linear optimization problems. They work by iteratively moving through the interior of the feasible region, converging towards optimal solutions. These methods strike a balance between exploration and exploitation, making them highly efficient in reaching near-optimal or optimal solutions. The simplicity and robustness of Interior Point methods have made them a popular choice in various fields, including engineering, economics, and machine learning.

In the context of controller tuning, Interior Point methods offer an effective approach to optimize the controller parameters, ensuring that the system performs in accordance with the desired specifications.

4. SIMULATION RESULTS

In the pursuit of enhancing the precision and efficiency of two-link robot arm systems, the

development of advanced control strategies holds paramount importance. This section presents the simulation results obtained from the implementation of our designed optimal MPID controller for a two-link robot arm. These simulation outcomes are a testament to the effectiveness and robustness of our control approach.
Throughout this section, we will delve into the simulation methodology, the establishment of control scenarios, and an in-depth analysis of the achieved results. These outcomes provide essential insights into the controller response, tracking accuracy, and overall system performance. The Simulink representation of a two-link robotic arm under the control of MPID controller is illustrated in Fig 3.

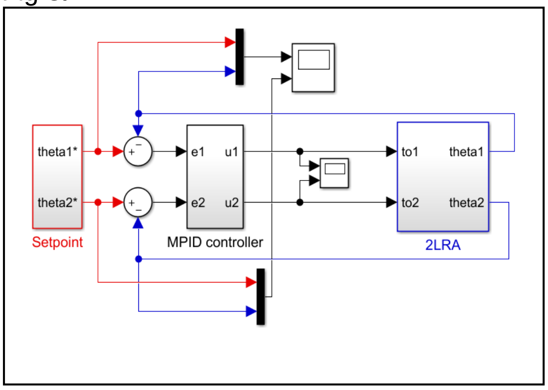


Fig. 3Simulink model.

Specifying Design Variables

Identify the controller parameters or system configurations you wish to optimize. These variables could include gains, time constants, filter coefficients, or any other parameters that affect the system behavior. Determine any constraints or limitations on the design variables. For example, you may have bounds on parameter values or constraints on system performance specifications, such as settling time, overshoot, or robustness requirements. In our case, we chose to restrict the minimum of the MPID coefficients to 0 because using negative values would complicate our study and may impact the stability of the closed loop system. Additionally, the maximum MPID coefficients value is limited to 300 to avoid saturation on the control signal.

Defining the Objective Function

Define the optimization objective function, which outlines the desired goal for the optimization process. This objective can involve minimizing or maximizing specific performance metrics, reducing errors, or achieving a particular system response. In this study, we aim to minimize the Integral of Time-weighted Absolute Error (ITAE). The ITAE is calculated for both outputs, θ_1 and θ_2, then they are added together to create a single minimization criterion.

$$Objectivefunction = \int (|e_1| + |e_2|) \cdot t \cdot dt \quad (8)$$

Running the Optimization

Execute the optimization process using the specified settings and algorithm. Simulink Response Analysis and Optimization Tool will automatically explore the design space, evaluating different combinations of the design variables to find the optimal solution. Fig. 4 represents the objective function values for each iteration.

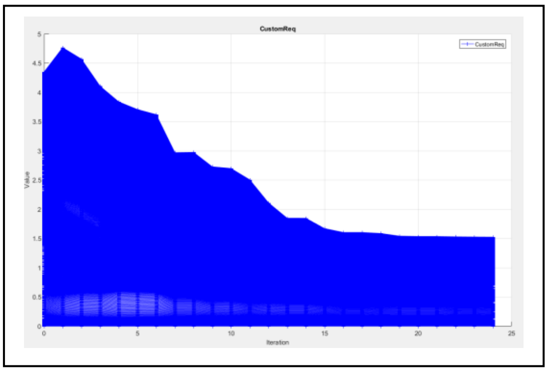
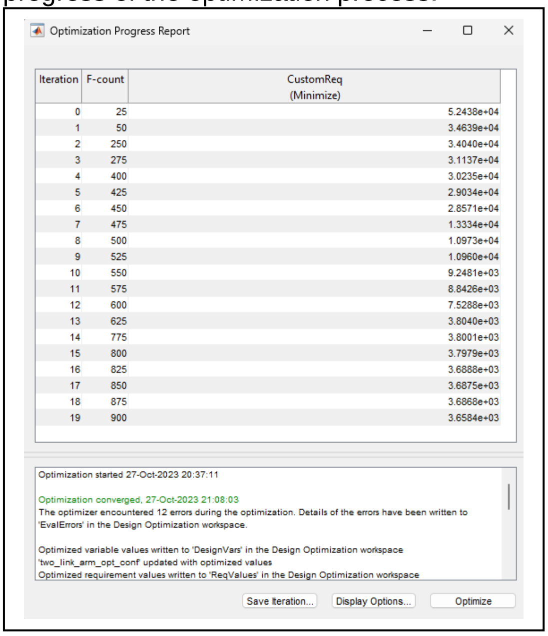


Fig. 4Objective function values for each iteration.

Figure 5 displays a screenshot of the report generated by MATLAB, which details the progress of the optimization process.


Optimization Progress Report

Iteration	F-count	CustomReq (Minimize)
0	25	5.2438e+04
1	50	3.4639e+04
2	250	3.4040e+04
3	275	3.1137e+04
4	400	3.0235e+04
5	425	2.9034e+04
6	450	2.8571e+04
7	475	1.3334e+04
8	500	1.0973e+04
9	525	1.0960e+04
10	550	9.2481e+03
11	575	8.8426e+03
12	600	7.5288e+03
13	625	3.8040e+03
14	775	3.8001e+03
15	800	3.7979e+03
16	825	3.6888e+03
17	850	3.6875e+03
18	875	3.6868e+03
19	900	3.6584e+03

Optimization started 27-Oct-2023 20:37:11

Optimization converged, 27-Oct-2023 21:08:03
The optimizer encountered 12 errors during the optimization. Details of the errors have been written to 'EvalErrors' in the Design Optimization workspace.

Optimized variable values written to 'DesignVars' in the Design Optimization workspace
'two_link_arm_opt_conf' updated with optimized values
Optimized requirement values written to 'ReqValues' in the Design Optimization workspace

Save Iteration... Display Options... Optimize

Fig. 5The results of each iteration.

The obtained optimal MPID coefficients are given in Tab. 1.

Table 1OPTIMAL MPID COEFFICIENTS

Coefficient	**Value**
Kp11	209.4690278660307
Kp12	98.15690368742716O
Kp21	64.639139959427510
Kp22	299.9712049295235
Ki11	299.9973444779502
Ki12	10.337465669529452
Ki21	101.3760318879350
Ki22	0.003999895020345
Kd11	41.516380751417260
Kd12	6.831102019012417
Kd21	14.009965807554632
Kd22	19.036589475151985

The results of simulation

To show the efficiency of the proposed approach, some simulation results are given. For the purpose of simulation, we assumed that the mass and the length of both the first and the second joints of the two-link robot arm are 1, i.e. M1 = 1 kg, M2 = 1 kg, L1 = 1 m, L2 = 1 m).

The initial and desired joint angles are given in Fig. 6 together with the real joint angels (output values).

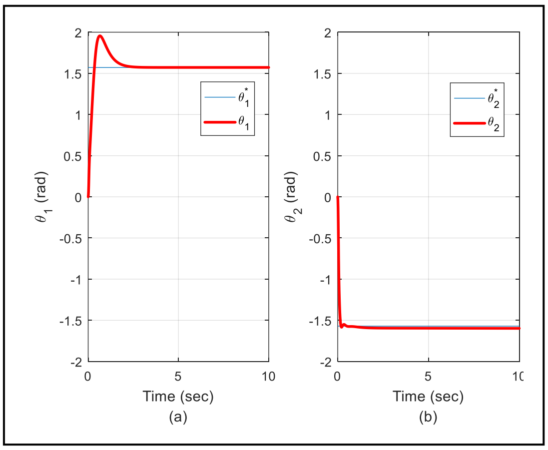


Fig. 6Desired and real orientations: (a) first link, (b) second link.

Fig. 7 and Fig. 8 represents the control input signal to the two-link robot arm, and the error signal between desired and actual joint angles, respectively.

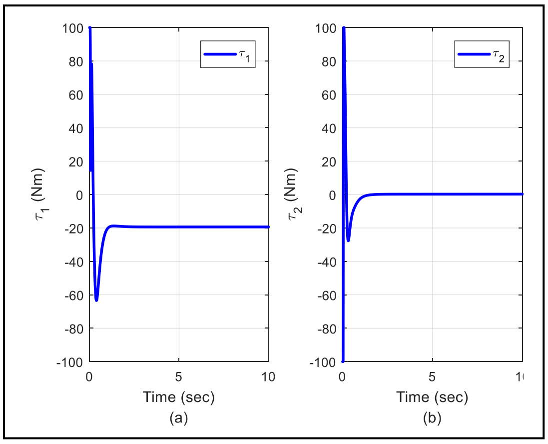


Fig. 7Robot torques: (a) first link, (b) second link.

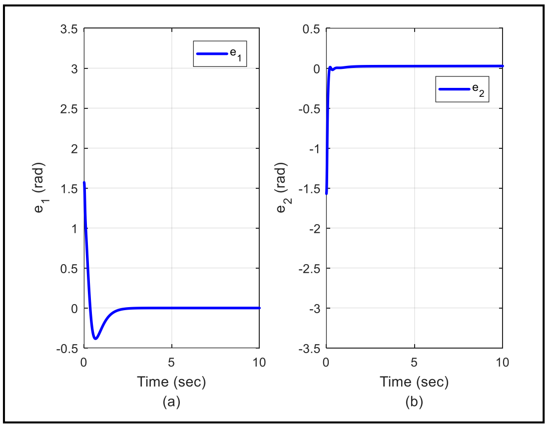


Fig. 8Errors of the joint angles: (a) first link, (b) second link.

In Fig. 6, it is evident that the optimized gains have significantly improved the settling time of the two-link robot arm. The arm rapidly reaches its desired position and settles down without any noticeable residual vibrations. As shown in Fig. 7, the torque profiles exhibit normal and expected behaviour when the joints reach their target positions. Fig. 8 reveals that the steady-state errors of the robot arm have been minimized to the extent that they can be safely disregarded. This implies that the two-link robot arm can achieve and maintain its desired position with a higher level of precision, thereby reducing any deviation from the target position.

5. CONCLUSIONS

In conclusion, the optimization of gains has effectively stabilized the robot arm, effectively reducing oscillations and overshoot in its response. This leads to heightened accuracy and enhanced reliability when performing tasks and reaching desired positions.

The response plots of the 2-link robot arm, stabilized through the MPID gains optimization method, serve as compelling evidence of the approach effectiveness in improving the two-link robot arm performance. This optimization significantly enhances stability, minimizes errors, and ensures precise control over the two-link robot arm motion. These findings underscore the promising potential of this approach in the field of robotic control.

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