Computationally Efficient Estimation of Heart Rate from PPG Signals

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Abstract: Heart rate (HR) monitoring using photoplethysmographic (PPG) signals is becoming increasingly popular due to the simplicity of device design and the low cost of wearable technology. Measuring HR from PPG signal using battery-powered wearable devices with limited resources requires computationally efficient algorithms. This paper presents a simple yet accurate heart rate estimation method based on a systolic peak detection algorithm that employs a fractional-order calculus-based filter. The proposed method was validated on the CapnoBase benchmark dataset, demonstrating an overall Average Absolute Error (AAE) of 1.78. These results are comparable to those achieved with state-of-the-art methods, while the proposed method offers the advantage of simpler implementation.

Keywords: heart rate, peak detection, PPG signal, bandpass differentiator.

1. INTRODUCTION

Heart rate (HR), defined as the number of heart beats per minute, is a vital physiological parameter that reflects cardiovascular health and fitness levels. Its accurate real-time measurement is essential for continuous health monitoring, even outside clinical settings. Traditionally, the electrocardiogram (ECG) signal has been regarded as the gold standard for measuring heart rate [1]. The ECG signals offers precise and reliable HR by recording the electrical activity of the heart. The precise HR measurement requires an accurate QRS complex detection. However, ECG has several drawbacks, particularly when it comes to continuous, long-term monitoring. The attachment of electrodes to the skin can be cumbersome and uncomfortable, often leading to skin irritation or allergic reactions. Additionally, the need for conductive gels, precise electrode placement, and wired connections can restrict patient mobility and convenience. These limitations make ECG less suitable for continuous use in everyday settings and for monitoring in ambulatory or home environments.

In recent years, photoplethysmography (PPG) signal has emerged as a promising alternative to ECG due to its non-invasive nature and ease of use [2]. The photoplethysmographic signal represents changes in blood volume in peripheral tissues, and is measured as the variations in the intensity of light (either reflected or transmitted) emitted by a light source (LED) and detected by a photo-detector [3]. PPG signals can be acquired at different locations of the human body including fingertips and earlobes for pulse oximetry and wrists for wearable devices such as smartwatches [4]. PPG signals can also be acquired through camera-based photoplethysmography methods that utilize smartphone cameras or webcams.

Unlike electrocardiography, which requires conductive gels, precise electrode placement, and wired connections [5], photoplethysmography is an optical technique that uses a simpler setup. This makes it more suitable for integration into wearable devices like smart-watches and fitness trackers, enabling continuous heart rate monitoring in everyday situations.

Despite its advantages mentioned above, the use of PPG signal for heart rate measurement is not without challenges. Factors such as motion artifacts, and ambient light interference can significantly affect the accuracy of the HR from the PPG signal. Furthermore, the PPG signal is susceptible to noise from different sources, including physical movements and changes in ambient light conditions, which can complicate the extraction of points like systolic peaks for accurate heart rate data. Therefore, sophisticated signal processing techniques and algorithms are required to extract reliable heart rate data from the noisy PPG signal.

The goal of this work is to address the aforementioned challenges by applying a digital filter based on fractional order calculus to enhance the accuracy and robustness of heart rate measurement using PPG signals. Specifically, the focus is on applying a

bandpass differentiator designed using fractional order calculus to accurately detect systolic peaks in the PPG signals, which will be used to measure HR. Fractional order calculus, which extends the concept of integer-order derivatives and integrals to non-integer orders, offers greater flexibility in the design of digital filters [6].

The core component of a heart rate estimation method based on the PPG signal is the beat detector. The more accurate the detector, the more reliable the heart rate measurements. The heart rate is often deduced from the inter peak intervals calculated as the time intervals between successive systolic peaks [7]. Previous methods developed to calculate heart rate from PPG signals have used various approaches, including time-domain peak detection [8], spectral-domain peak detection [9], and machine learning algorithms and deep learning models [10]. Although these methods have been successfully applied to heart rate measurements, they are computationally expensive and/or sensitive to noise.

Measuring HR from PPG signal using battery-powered wearable devices with limited resources requires computationally efficient peak detectors . In this work, heart rate is estimated using PPG systolic peaks, which are detected using a computationally efficient and accurate algorithm for PPG systolic peak detection. The proposed method is based on a bandpass differentiator designed using fractional order calculus which can perform differentiation and noise removal in the same operation. The inflection points in the PPG signal are highly enhanced in the bandpass differentiator output even in the case of very noisy PPG signals, making it possible to accurately detect systolic peaks, and thereby improving the reliability of PPG-based heart rate measurements. The remainder of the paper is structured as follows: materials and methods are described in Section 2. In Section 3, we report the results of the proposed method. In Section 4, we demonstrate synthesis results and compare them with those generated by previous approaches. In Sections 5, we provide the conclusion.

2. MATERIALS AND METHODS

A. Proposed method

The proposed method for heart rate estimation from PPG signal is depicted in Fig. 1. It makes use of the PPG systolic peak detection algorithm described in our previous work [11].

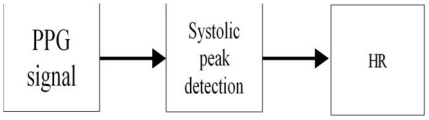

Fig. 1 Block digram of the proposed method.

This algorithm is based on a bandpass differentiator designed using fractional order differentiation with negative order to prevent noise amplification during the process of detection of maxima and inflection points in the signal. It consists of four stages: bandpass differentiation, half wave rectification, signal squaring, and decision rule module. Unlike many other peak detectors, this method applies the raw PPG signal directly to the peak detector without any preprocessing.

band-pass differentiation

This operation is performed by a bandpass differentiator whose output is the difference of forward and backward truncated temporal fractional derivative for a given fractional order α. Its output is given by:

$$y(n) = y_f(n) - y_b(n) \quad (1)$$

$$y(n) = \sum_{i=0}^{\infty} h_\alpha(i)x(n+i) - \sum_{i=0}^{\infty} h_\alpha(i)x(n-i) \quad (2)$$

with

$$h_\alpha(i) = -\frac{a_i}{T_S^\alpha}, i = 1,2,3,\ldots \quad (3)$$

Where $x(n)$ are samples of the PPG signal sampled with period T_s, α is the fractional order, and a_i are the expansion coefficients of the series$(1-z^{-1})^\alpha$. The coefficients a_i can be computed recursively by:

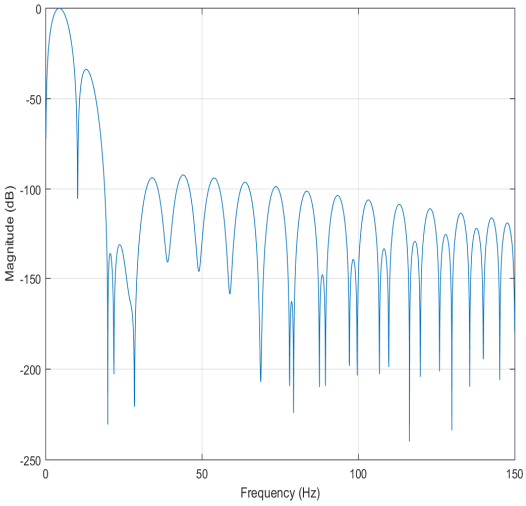

Fig. 2 Amplitude frequency responses of the bandpass differentiator for M=31and α=-0.9 used for systolic peak detection in PPG signals sampled at 300 Hz.

$$a_0 = 1,\ a_n = \frac{n-\alpha-1}{n}\, a_{n-1}, n = 1,2,3,\ldots \quad (4)$$

This infinite impulse response is truncated between – (M – 1)/2 and (M – 1)/2, where M is an odd integer. This truncation leads to a non-causal FIR filter whose amplitude response is given by:

$$H_{\alpha M}(jf) = 2j \sum_{i=1}^{(M-1)/2} h(i).sin(2f\pi iT_s) \quad (5)$$

After truncation, the frequency response depends on the parameter M and the fractional order α. Figures 3 and 4 shows the frequency response of the bandpass differentiator filter for different value of M and α.

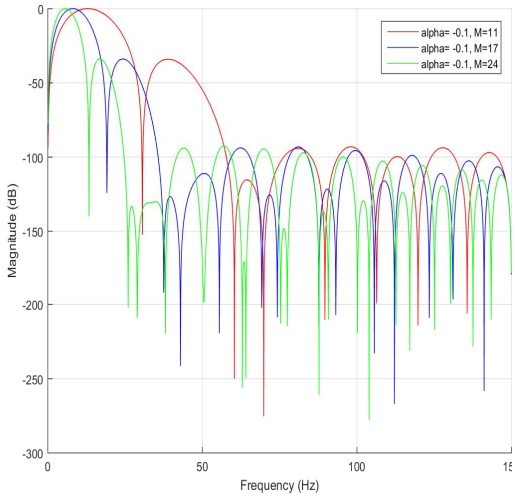

Fig. 3. Amplitude frequency response of the bandpass differentiator for different values of the parameter M and the fractional order $\alpha = -0.1$.

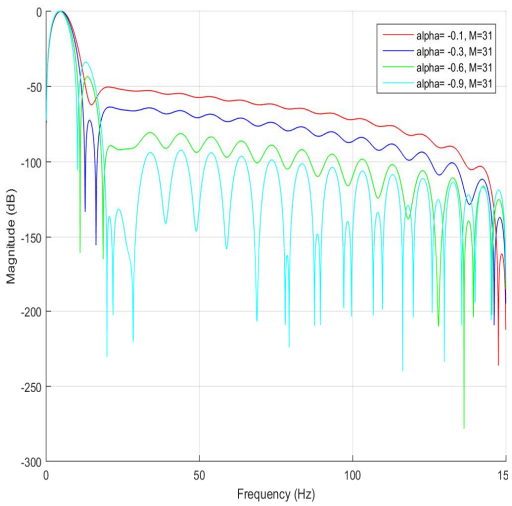

Fig. 4. Amplitude frequency response of the bandpass differentiator for different values of the fractional order α and the parameter $M = 31$.

Figure 2 shows the amplitude frequency response of the bandpass differentiator for M=31 and α=-0.9. The low cut-off frequency and high cut-off frequency measured at -3 dB are 2.78 Hz and 5.85 Hz, respectively.

Half wave rectification

The half-wave rectification part of the algorithm removes the negative half of the band-pass differentiator output signal. Its purpose is to prepare the signal for the search of systolic peaks by removing what is considered as unwanted components that hinder accurate systolic detection. Replacing negative values with zeros will also reduce the computation time, making the algorithm more suitable for resource-limited devices like smartwatches.

Signal squaring

Sometimes the filtered and rectified PPG signal may exhibit more than one positive peak with different amplitudes. This will influence the search of the systolic peak. To reduce this impact, after the rectification phase the signal is subjected to a squaring process which leads to make a huge difference in value between these peaks, and therefore enables the use of very small adaptive threshold for more accuracy [12]. Figure 5 shows the results of the different systolic peak algorithm detection

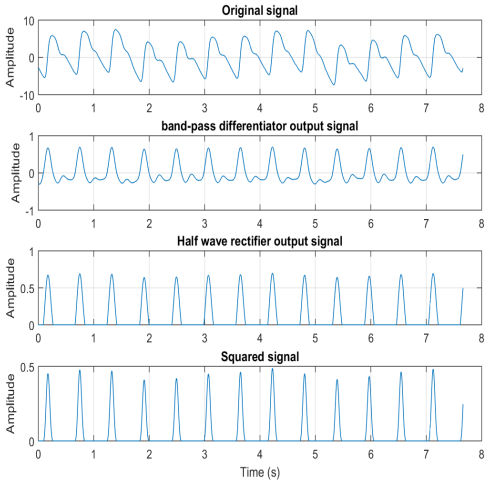

Fig. 5 Results of the different systolic peak algorithm detection phases.

phases.

Decisions rules

In this stage of the algorithm, the squared signal is examined for points exceeding a predefined threshold within a window of length 100 ms. If such a point is identified, the algorithm then searches for the corresponding systolic peak within a window of 300 ms in the

raw PPG signal. The systolic peak is determined as the maximum value within this window. The index of this peak is recorded, and a refractory period of 400 ms is applied to prevent false detections. Figures 6 and 7 shows the results of application of the systolic peak detection algorithm to various PPG signals along with the reference systolic peaks. The vertical line represents the annotated systolic peak, while the 'O' symbol denotes the peak detected by the algorithm. One can

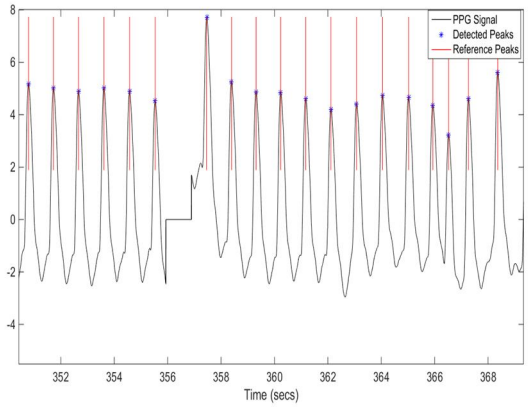

Fig. 6 Systolic peak detection by the proposed algorithm. Record 0309_8min.mat.

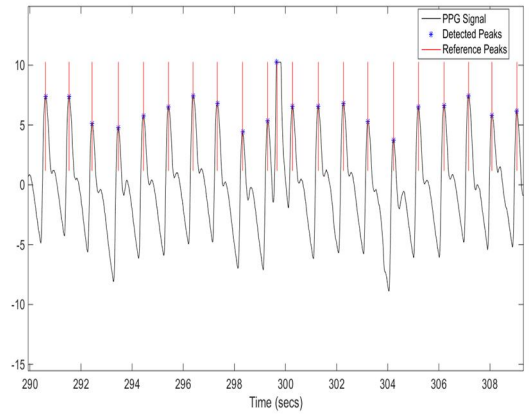

Fig. 7 Systolic peak detection by the proposed algorithm. Record 0150_8min.mat.

observe a good agreement between the detected peaks and the reference peaks.

In some records, we observed that the PPG signal exhibits saturation (where the output signal remains constant at its maximum) during recording, leading to signal distortion. Consequently, the expert, when setting the reference peak, assumes that the actual peak occurs in the middle of the saturation period as shown in Fig.8. Also in some cases, the expert places the reference peak indicating the maximum of the signal far away as shown in Fig. 9.

B. Dataset

The PPG dataset used in this study is the benchmark CapnoBase dataset [13]. This dataset, which is publically available at: https://borealisdata.ca/dataverse/capnobase. Contains 42 eight-minute recordings collected from 42 patients (29 children and 13 adults) during elective surgery and routine anesthesia. These recordings were sampled at 300 Hz. The dataset also contains ECG and capnography signals. Labels from expert annotators are available for PPG peaks breaths from capnography.

C. Heart rate estimation

To calculate the heart rate from the raw PPG signal, we applied the PPG systolic peak detector developed in our previous work [11] to determine the peak-to-peak intervals. The instantaneous heart rate is then calculated using the following formula:

$$HR = \frac{60}{P_i - P_{i-1}} \ (bpm) \qquad (6)$$

where $P_i - P_{i-1}$ is the peak-to-peak interval in seconds, and the acronym 'bpm' stands for beats per minute. Instantaneous heart rate is frequently used in heart rate variability (HRV) studies to evaluate overall heart health and the functioning of the autonomic nervous system, which controls cardiac activity. $HR_e(j)$ and $HR_t(j)$ denote the estimated HR by the proposed algorithm and the reference HR from the dataset at the j-th time, respectively.

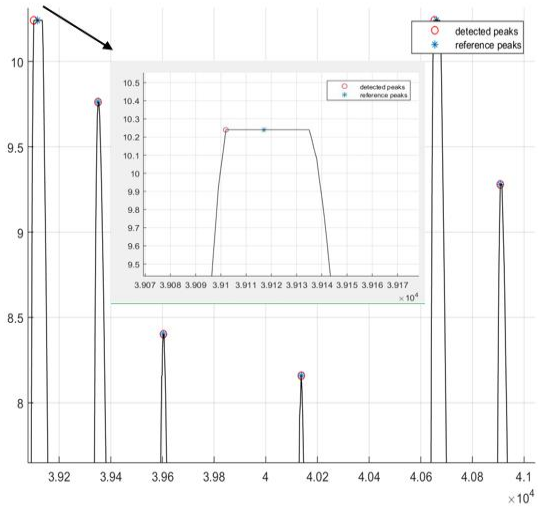

Fig. 8 Saturation while recording raw PPG signal.

Table 1 Performance of HR estimation on the CapnoBase dataset.

Record name	0009_8min.mat	0015_8min.mat	0016_8min.mat	0018_8min.mat	0023_8min.mat	0028_8min.mat
AAE	0	0	4.37	2.19	0	0
Record name	0029_8min.mat	0030_8min.mat	0031_8min.mat	0032_8min.mat	0035_8min.mat	0038_8min.mat
AAE	1.61	5.10	4.55	1.86	4.62	8.43
Record name	0103_8min.mat	0104_8min.mat	0105_8min.mat	0115_8min.mat	0121_8min.mat	0122_8min.mat
AAE	2.47	1.00	2.86	7.78	0	0
Record name	0123_8min.mat	0125_8min.mat	0127_8min.mat	0128_8min.mat	0133_8min.mat	0134_8min.mat
AAE	3.29	0	2.50	0	4.57	3.33
Record name	0142_8min.mat	0147_8min.mat	0148_8min.mat	0149_8min.mat	0150_8min.mat	0309_8min.mat
AAE	1.81	1.18	1.75	0.4	1.48	1.25
Record name	0311_8min.mat	0312_8min.mat	0313_8min.mat	0322_8min.mat	0325_8min.mat	0328_8min.mat
AAE	0.71	2.30	0	0.99	0.22	1.29
Record name	0329_8min.mat	0330_8min.mat	0331_8min.mat	0332_8min.mat	0333_8min.mat	0370_8min.mat
MAE	0	0	0	0	0.82	3.89
Over all AAE	**1.78**					

3. RESULTS

To demonstrate the effectiveness of the proposed method, some experiments are conducted, and the results are reported and compared with those of existing methods. To evaluate the performance of the method for heart rate estimation, the Average Absolute Error (AAE) is used. AAE is defined as:

$$AAE = \frac{1}{N} \times \sum_{j=1}^{N} AE(j) \quad (7)$$

where AE is the Absolute Error defined as:

$$AE(j) = |HR_e(j) - HR_t(j)| \quad (8)$$

Table 1 presents the Average Absolute Error (AAE) for the 42 records. The AAE values show considerable variability across different records, ranging from 0 to 8.43. However, the majority of AAE scores fall within the 0-3 range, with an overall AAE of 1.78. Figure 10 shows an example of the reference and estimated heart rate for the recording "0311_8min.mat".

4. COMPARISON WITH EXISTING METHODS

Many methods for heart rate estimation have been evaluated on various PPG datasets, and their performance is highly dependent on the dataset used. A recent paper [14] reports an Average Absolute Error (AAE) of 1.05 on the CapnoBase dataset. In comparison, the proposed method achieves an AAE of 1.78, while offering the advantages of simpler implementation and lower computational cost.

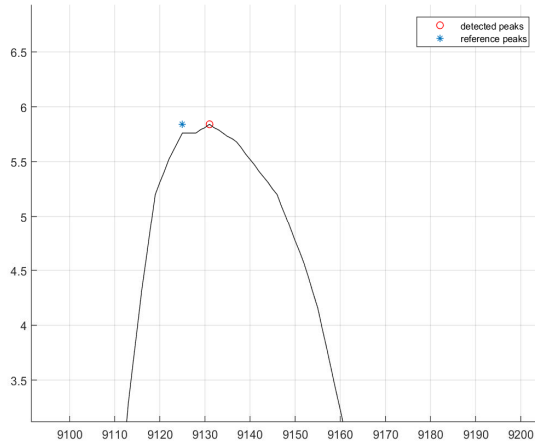

Fig. 9 Incorrectly placed reference peak.

An MAE of 5 beats per minute (bpm) might be acceptable in some contexts, but in others, such as precise medical diagnostics or high-performance sports monitoring, an MAE of 1 bpm or less might be required. This study shows an MAE of 2.75 bpm, which is a very good result according to the study in [15].

5. CONCLUSION

An efficient and accurate method using a fractional order calculus based bandpass differentiator has been proposed for heart rate estimation from PPG signal and it is well-suited for real-time and resource-constrained applications, such as wearable health monitoring devices. Future work will focus on enhancing its robustness against motion artefacts and evaluating its performance in dynamic settings, particularly during physical activity.

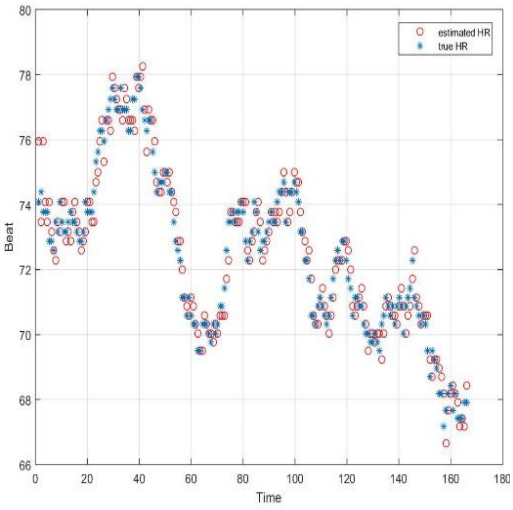

Fig. 10 The true and estimated heart rate for recording 0311_8min.mat.

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