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Dependability Measures Estimation Using Binomial Failure Rate Model

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Abstract: Dependability of multi-component systems is highly impacted by common cause failures, what necessitates the appropriate consideration of such events in the dependability modeling process. This paper is dedicated to study the application of the binomial failure rate model in handling the contribution of common cause failures to estimate two key dependability indicators, namely: unavailability and unconditional failure intensity, using fault tree analysis with the probabilistic treatment of the associated parameter uncertainty. The results of such application are thoroughly compared to those of the traditional Beta factor model to highlight the possible differences.

Keywords: Dependability, Common Cause Failures, Beta Factor Model, Binomial Failure Rate Model.

1. INTRODUCTION

Monitoring and controlling the dangerous behavior of the industrial processes, which are often characterized by their high level of intricacy, necessitates the implication of several kinds of lines of defense that match such complexity and riskiness. However, since the appropriate operation of such safety systems is not guaranteed, their individual and collective performance must be carefully assessed through the consideration of the possible jeopardizing events and conditions.

As a key contributor in the malfunctioning of the safety systems that hold redundant components, the matter that led at several opportunities to tragic events (e.g., Three Mile Island), common cause failures (CCF) represent a significant issue that must be included in dependability and risk analyses involving systems subject to such failures. This type of events are defined in [1] as dependent failures in which two or more component fault states exist simultaneously, or within a short time interval, and are a direct result of a shared cause.

As highlighted in [2], modeling CCF is being acknowledged as one of the most demanding issues in the probabilistic safety assessments due to their multilevel specificity, the matter that initiated several efforts in purpose of suitably handling many associated issues. Recently for instance, a new model for CCF is developed in [3] considering components degradation, while [4] focused on using field experience from the petroleum industry to enhance the involved data.

Actually, such modeling task can be carried out by means of several alternative models that differ in terms of sophistication and requirements, among which we find the so-called “parametric models”. A detailed description of a set of this kind of models can be found in [5, 6]. Subsequently, two of the most commonly used ones are briefly described.

1.1. Beta Factor model

First introduced in [7], the Beta Factor (BF) model belongs to the non-shock category of the parametric models and it involves one single factor denoted by β. This factor represents the proportion of CCF events that can affect more than one component in a defined common cause component group (CCCG) of size m.

According to this model, the failure rate of a set of k components in a given CCCG can be expressed as follows:

$$\lambda_k^m = \begin{cases} (1-\beta)\lambda_t = \lambda_I & k=1 \\ 0 & 1<k<m \\ \beta\lambda_t & k=m \end{cases} \quad (1)$$

where: λ_t represents the total failure rate of each component, and λ_I is the independent failure rate.

Despite the simplicity of this traditional model, its assumption that the occurrence of a CCF event will inevitably cause the failure of the whole CCCG, the matter that is explicitly shown in (1), is commonly viewed as an obstruction for the applications where the CCCG involves more than two components.

Several models have been proposed as alternatives, like for example: the Alpha Factor model [8], Multiple Greek Letter model [9], Multiple Beta Factor model [10] and Binomial Failure Rate model [11].

1.2. Binomial Failure Rate model

The Binomial Failure Rate (BFR) model is a shock model in which failures are originated to independent or shock causes. Furthermore, the shock ones are classified into lethal failures which can affect all the *m* components, and non-lethal failures which can affect any number of components in the CCCG with binomially distributed probability. Hence:

$$\lambda_k^m = \begin{cases} \lambda_I + \rho\mu(1-\rho)^{m-1} & k=1 \\ \mu\rho^k(1-\rho)^{m-k} & 1<k<m \\ \mu\rho^m + \omega & k=m \end{cases} \quad (2)$$

where: ρ denotes the conditional probability of failure of each component given a non-lethal event, μ represents the occurrence rate of the non-lethal events, while ω is the occurrence rate of the lethal shocks.

As the traditional one, BF model is the commonly used parametric model in different sectors and applications, like the international standard IEC-61508 entitled “Functional safety of electrical/electronic/programmable electronic safety-related systems” which aims to provide a technical approach for ensuring an appropriate employment of such safety systems from the initial concept, though design, implementation, operation and maintenance until their decommissioning.

As part of their design, the IEC-61508 standard requires to carry out dependability assessments of the safety-related systems’ (SRS) performance in order to ensure their ability to fulfill their assigned safety functions. For this end, the contribution of CCF events is taken into account using BF model with a suggested methodology to estimate its involved parameter.

However, in the current version [12] and besides that traditional model, it suggests in its sixth part to employ the BFR model in order to avoid the conservatism problem related to the cases of having more than double failures.

The main purpose of this work is applying the BFR model to estimate the two main SRS performance indicators that are considered in IEC-61508, namely: a) the average probability of dangerous failure on demand (PFD_{avg}) which represents the average unavailability, and b) the average frequency of a dangerous failure (PFH) which represents the average unconditional failure intensity (also known as “failure frequency”) using fault tree analysis. The results of such model are then compared to the those obtained by means of the traditional BF model.

2. IMPLEMENTATION

Let us consider a subsystem of a safety-related system composed of four identical components (see Fig.1). To study the application of these two parametric models, and analogously to the implementation of the IEC-61508 standard for the BFR model, we will only consider the contribution of the dangerous undetected failures, with the failure rate λ_{DU}=7.61E-5 h^{-1}. Indeed such failures can only be detected during the proof test that is performed annually in this application (i.e., *T*=8760 *h*).

However, the same procedure can be followed when integrating the contribution of the dangerous detected failures.

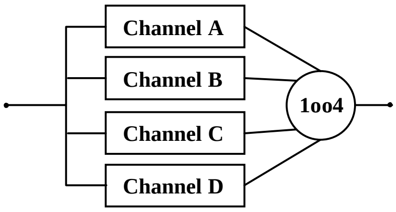

Fig. 1 1oo4 subsystem

2.1. Implementation of BF model

As an illustration, we employ at this stage the BF model to take into consideration the contribution of CCF in the estimation of PFD_{avg} and PFH of the studied subsystem which represent the main dependability measures for both low demand mode and high/continuous demand modes respectively. Figure 2 shows such implementation in a fault tree model.

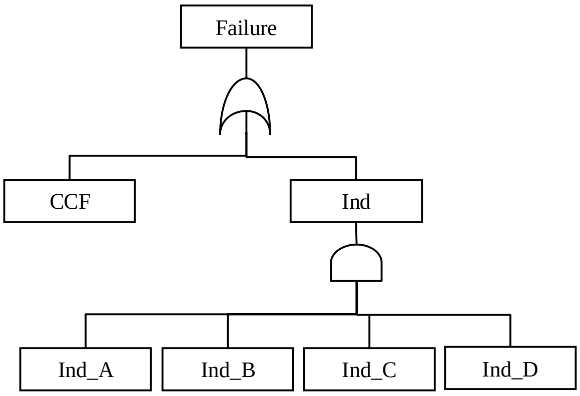

Fig. 2 Fault free related to the 1oo4 subsystem using BF model

By using β = 0.197, the quantitative analysis of this fault tree gives a value of 6.92E-2 for PFD_{avg} and 1.70E-5 h^{-1} for PFH.

2.2. Implementation of BFR model

We use now the BFR model instead of the traditional BF model to estimate the same dependability measures. The related fault tree is given in Fig. 3.

This time the employed parameters are: λ_I=6.11E-5 h^{-1}, ρ= 0.4, μ=1.83E-5 h^{-1} and ω=7.69E-6 h^{-1}.

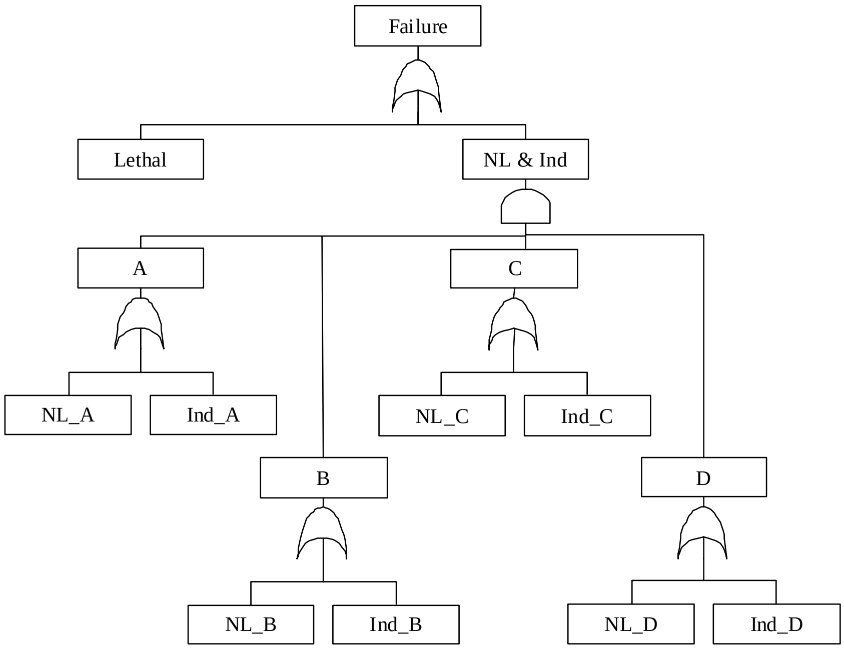

Fig. 3 Fault free related to the 1oo4 subsystem using BFR model

The obtained results are PFD_{avg}=4.25E-2 and PFH=1.18E-5 h^{-1}. Obviously, and as explicitly shown in Figs. 4 and 5, these latter values are lower than those obtained using BF model.

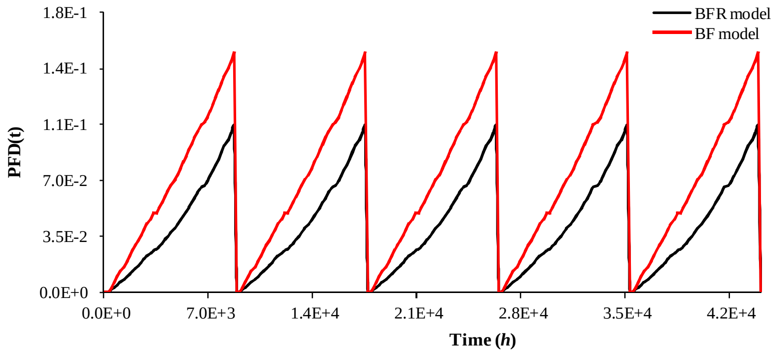

Fig. 4 Obtained PFD(t) using BFR and BF models

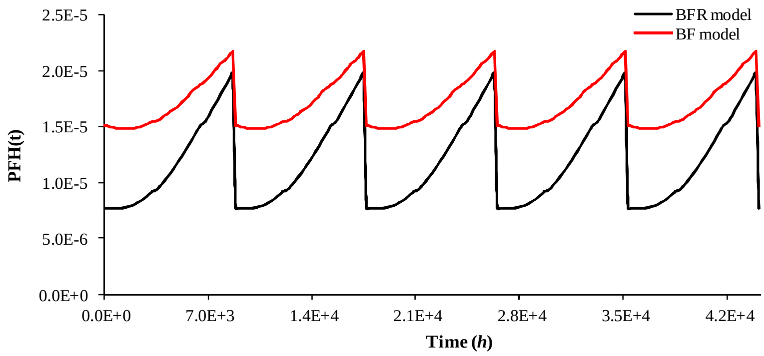

Fig. 5 Obtained PFH(t) using BFR and BF models

In order to further investigate the differences between the two studied parametric models we apply them again for the rest of architectures that hold four components, namely: 2oo4, 3oo4 and 4oo4 to estimate the same performance indicators.

The obtained results for those configurations as well as the first treated one are gathered in Table 1, which show that, in contrast to 1oo4 and 2oo4 architectures, the BFR model is providing values that are higher than those given by means of BF model for the 3oo4 and 4oo4 architectures.

Table 1 PFD_{avg} and PFH values for the Moo4 architectures using BF and BFR models

Architecture	**PFD_{avg}**		**PFH (h^{-1})**	
	BF	**BFR**	**BF**	**BFR**
1oo4	6.92E-2	4.25E-2	1.70E-5	1.18E-5
2oo4	1.19E-1	1.08E-1	3.37E-5	3.33E-5
3oo4	2.87E-1	3.01E-1	6.90E-5	7.25E-5
4oo4	6.05E-1	6.28E-1	1.03E-4	1.05E-4

However, it is important to note that such implementation of the BFR model is simplified and the complete fault tree model of the studied subsystem that considers all the possible combinations gives slightly different results, particularly in the case of PFD_{avg} of the 1oo4 architecture as depicted in Table 2.

Table 2 PFD_{avg} and PFH values for the Moo4 architectures using BFR model by the complete fault tree model

Architecture	**PFD_{avg}**	**PFH (h^{-1})**
1oo4	4.79E-2	1.32E-5
2oo4	1.18E-1	3.44E-5
3oo4	3.00E-1	7.12E-5
4oo4	6.14E-1	1.03E-4

3. UNCERTAINTY AND SENSITIVITY ANALYSIS

In this section, uncertainty associated with the involved input parameters is taken into account in the estimation of the two performance indicators of the 1oo4 subsystem using Monte Carlo simulation for both parametric models. The outputs' sensitivity to such uncertain quantities is also studied.

The utilized data for both parametric models is gathered in Table 3.

Table 3 Reliability data

Parameter		**Data**
BFR model	λ_I (h^{-1})	LogNormal (-9.801, 0.44)
	ρ	Uniform (0.2, 0.6)
	μ (h^{-1})	LogNormal (-11.861, 1.38)
	ω (h^{-1})	LogNormal (-12.0205897, 0.7)
BF model	λ_t (h^{-1})	Lognormal (-9.68191, 0.63)
	β	Uniform (0.01, 0.384)
T (h)		8760

The propagation of those quantities through the associated models with a number of 1E5 runs yielded the results of Table 4 and Figs. 6 and 7. Obviously, the mean values of both PFD_{avg} and PFH are slightly higher than the results of section 2 for both parametric models. However, the gap between the obtained values from the two models remains noticeable, especially in the case of PFD_{avg}.

Table 4 Uncertainty analysis

Model	PFD_{avg}		PFH	
	Mean	**Variance**	**Mean**	**Variance**
BFR	5.35E-2	1.23E-3	2.75E-5	9.46E-10
BF	7.39E-2	4.54E-3	3.07E-5	2.70E-9

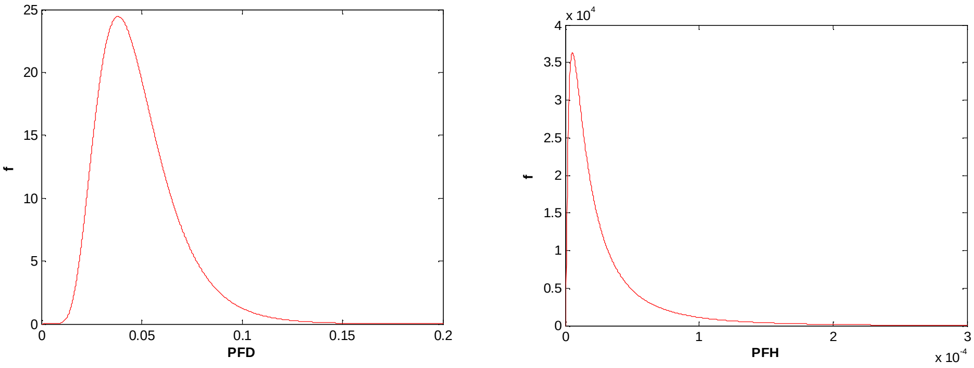

Fig. 6 Density functions of PFD_{avg} and PFH related to BFR model

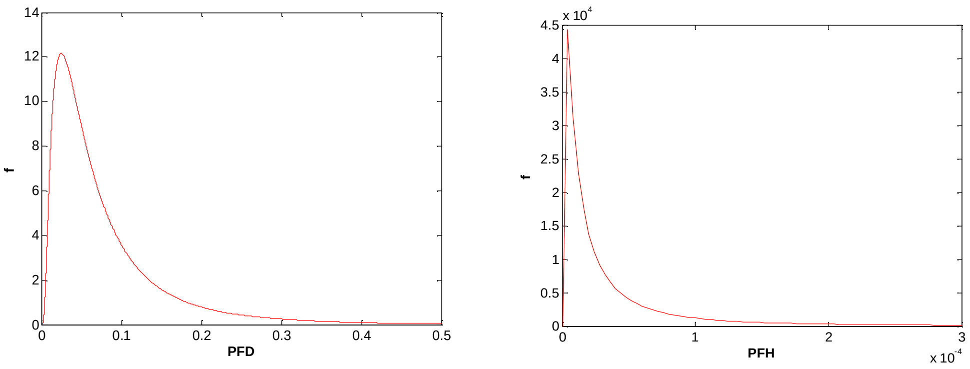

Fig. 7 Density functions of PFD_{avg} and PFH related to BF model

The sensitivity of the two studied performance indicators to the various uncertain parameters is analyzed by means of two variance-based indices, namely: Fourier amplitude sensitivity test (Fast) [13], and Sobol [14, 15].

The obtained results for both models are shown in Table 5. In the case of BF model, both indices reveal that both PFD_{avg} and PFH are primarily sensitive to λ_t. For BFR model, Fast shows the high sensitivity of PFD_{avg} to ω and λ_l respectively and indicates the dominance of λ_l for PFH (see Fig. 8), while according to Sobol index, the main contributors in the uncertainty related to PFD_{avg} are λ_l and μ, and that related to PFH is μ, as graphically depicted in Fig. 9.

Table 5 Sensitivity analysis

Parameter		Fast		Sobol	
		PFD_{avg}	PFH	PFD_{avg}	PFH
BFR model	λ_l	3.1E-1	4.5E-1	4.4E-1	4.3E-2
	ρ	1.8E-2	2.6E-2	0	0
	μ	1.0E-1	1.5E-1	3.8E-1	6.1E-1
	ω	3.7E-1	3.1E-2	1.8E-2	2.9E-2
BF model	λ_t	7.0E-1	5.6E-1	1.5E-1	1.1E-3
	β	1.5E-1	2.9E-3	0	0

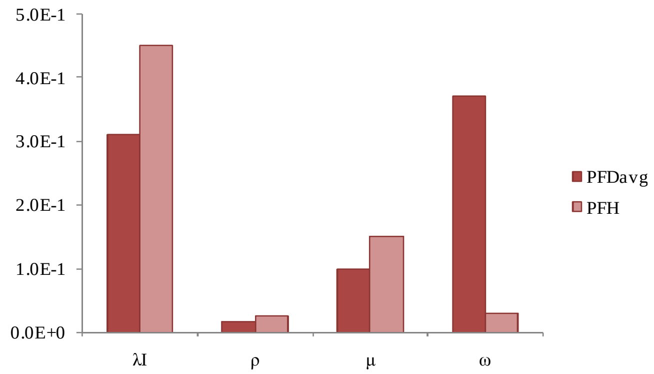

Fig. 8 Sensitivity analysis of PFD_{avg} and PFH related to BFR model using Fast index

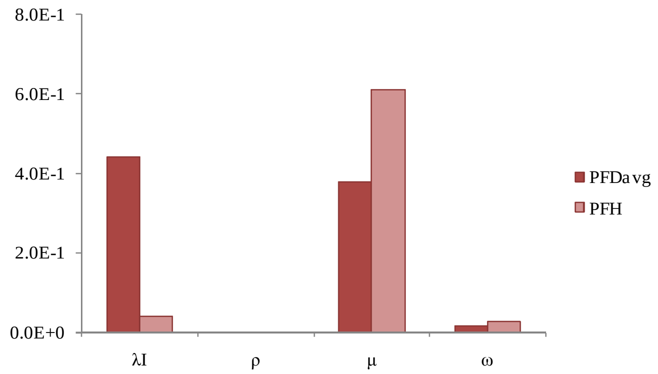

Fig. 9 Sensitivity analysis of PFD_{avg} and PFH related to BFR model using Sobol index

4. FURTHER EXAMINATION

IEC-61508 standard provides an example of a procedure that can be used if no data are available to feed the BFR model when there are more than 3 similar items, which is based on employing the β factor of the BF model, neglecting the lethal failures' proportion and estimating the remaining parameters in following manner:

$$\rho = \sqrt{\frac{C_N^2}{10C_N^4}} \tag{3}$$

$$\mu = \frac{\beta\lambda_{DU}}{C_N^2\rho^2 + C_N^3\rho^3} \tag{4}$$

where: C_N^2, C_N^3 and C_N^4 represent the number of the potential double, triple and quadruple failures of a number of *N* similar items.

By applying such procedure to the studied 1oo4 subsystem, we can find that PFD_{avg}=1.87E-2 and PFH=8.40E-6 h^{-1} which means that the employed procedure can underestimate the values of the two performance indicators.

5. CONCLUSION

Aiming to examine the possible advantages of BFR model over the traditional BF model in terms of modeling the contribution of CCF events in the estimation of PFD_{avg} and PFH, both parametric models are employed to analyze the dependability of a 1oo4 safety sub-system by means of fault tree analysis.

The obtained results show that for this specific architecture, BFR model is giving the lowest values in comparison with the other alternative for both PFD_{avg} and PFH, especially in the case of using the simplified fault tree model. However, for some other architectures that share this number of components, it is the opposite. Uncertainty analysis showed the slight underestimation that can be obtained if such consideration is neglected. Indeed, this negative impact can be more significant for the highly complex and uncertain practical systems. What is more, several elements may contribute to the accumulation of those tenuous underestimations, as it is shown in section 4, and lead to the hidden invalidity of the whole modeling process.

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