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Nonlinear Backstepping Control of Permanent Magnet Synchronous Motor with Rotor Speed and Position Estimation

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Abstract: This paper presents a nonlinear backstepping control strategy used to ensure good dynamic behavior, high performance and the stability of the permanent magnet synchronous motor (PMSM). However, this control requires the precise knowledge of certain variables (speed, torque and position) that are difficult to access or sensors require additional mounting space, reduce reliability, increase the cost of the engine, and make maintenance difficult. Thus, an Extended Kalman Filter (EKF) approach is proposed for the estimation of speed and rotor position in the PMSM. The interesting simulation results obtained which are subjected to the load perturbation show very well the efficiency and the good performance of the nonlinear feedback control proposed and simulated in Matlab-Simulink.

Keywords: Permanent magnet synchronous motor (PMSM), nonlinear backstepping control, sensorless control, Extended Kalman filter (EKF), Matlab-Simulink.

1. INTRODUCTION
PMSM is widely used in industrial applications compared to other electric motors. Mainly, due to its compact design, high efficiency, high torque to inertia ratio, excellent reliability, great robustness and reduced maintenance [1-4] PMSM is used in robotics, aircrafts, naval applications, servomotors, DVD drivers and in other many fields.

In other side, the non-linearity of the dynamic model of the PMSM produces a large specific control difficulty. Parameters and load torque variations also the coupling between the motor speed and the electrical quantities, such as the d-q axis currents, making this system obviously hard to control [1,3,4]. This motor can be controlled by the conventional PI classical controller but cannot guarantee satisfactory performance such as stability and control against disturbances [5]. To solve this problem, various methods of nonlinear control have been developed and proposed for the PMSM system, such as input-output linearization control [6,7], robust control [8,9], sliding mode control [10-12], backstepping control [13-15], fuzzy logic control [16] and DTC [17] ...etc.

Recently, the backstepping presents promisingly alternative methods for controlling nonlinear systems. Combining the choice of the Lyapunov function with the control laws, this allows at all times the overall stability of the system [14].

Since mechanic position and speed sensors are usually too expensive, increase the cost and complexity, decrease the stability of the system, impose hard maintenance and are sensitive to noise. Thus, sensorless control is becoming a research focus now and a big challenging task [18]. The state observer permits an accurate estimation of the speed and the position in the presence of measurement and system noise and parameters variations. There are many observers such as sliding mode observer [18], Luenberger [19], back-EMF [20], model reference adaptive system method (MRAS) [21]. Among those, the Extended Kalman filter (EKF) is the most well-known and optimal one, especially in the presence of the process and measurement noise [22,23], this estimator is well adapted to nonlinear systems.

In this paper, the proposed strategy consists of elaborating a backstepping control technique applied to the PMSM for the speed tracking reference using the EKF as a state estimator. This vise to introduce a sensorless control, which increases the robustness of the proposed controller-observer, ensures also the good dynamic, high performance, guarantees the stability of the PMSM and decreases the cost and complexity.

To achieve the proposed approach, more than the introduction this paper in organized in five sections. Second section is devoted to the dynamical model of the PMSM. The third one is for the backstepping control design. The nonlinear sensorless control is designed for the PMSM using EKF.
in section 4. The simulation results demonstrating the effectiveness of the proposed strategy are illustrated in section 5. Finally, a conclusion is drawn.

2. MATHEMATICAL MODEL OF THE PMSM CONTROL ESTIMATIONSPEED

2.1. Mathematical Model of the PMSM

The PMSM model in the (d–q) reference frame is given as [6, 7, 12]:

\[
\begin{align*}
\frac{di_d}{dt} &= -\frac{R_s}{L_d}i_d + \frac{1}{L_d}p\Phi_f \Omega + \frac{1}{L_d}V_d \\
\frac{di_q}{dt} &= -\frac{R_s}{L_q}i_q + \frac{1}{L_q}p\Phi_f \Omega + \frac{1}{L_q}V_q \\
\frac{d\Omega}{dt} &= \frac{3p\Phi_f}{2j}i_q + \frac{3p}{2j}(L_d - L_q)i_di_q - \frac{f}{j}\Omega - \frac{T_L}{j}
\end{align*}
\]  

(1)

Where:
- \( f \) Rotor inertia
- \( i_d, i_q \) Stator currents in (d–q) reference frame
- \( V_d, V_q \) Stator voltages in (d–q) reference frame
- \( T_L \) Load torque
- \( \Phi_f \) Magnet flux
- \( \Omega \) Rotor mechanical speed

Equation (2) represents the dynamic model of a nonlinear system with the general form as follow:

\[
\begin{align*}
\frac{dx}{dt} &= f(x) + g(u) \\
y &= h(x)
\end{align*}
\]  

(3)

2.2. Backstepping Control Design

Many works in literature reported on the stability of a virtual control state and Lyapunov stability theory for stabilizing error variable. The Backstepping technique guarantees the asymptotic stability of the system in close-loop [13, 15]. Furthermore, this controller is very efficient for nonlinear systems.

In fact, the dynamic model of PMSM is highly nonlinear due to the coupling between the system states. Then, in order to assimilate the PMSM to DC motor the vector control principle is introduced. Moreover, to orient all the linkage flux in the d axis and obtain the maximum torque, the \( i_d \) current is always forced to be zero [19].

The principal objective of the proposed backstepping controller is to ensure the reference speed tracking of the PMSM drive in the presence of external disturbances as the torque load [19].

The proposed robust controller is designed in two steps as follow:

2.3. Speed Controller Design Methodology

a). step1. The speed tracking error variable can be defined as

\[
e_{x_3} = x_{3_{ref}} - x_3
\]  

(4)

The derivative of the speed tracking error dynamic is given as follow:

\[
\dot{e}_{x_3} = \dot{x}_{3_{ref}} - \dot{x}_3
\]  

(5)

From (2) and (5), the speed error derivative becomes:

\[
\dot{e}_{x_3} = \dot{x}_{3_{ref}} - \frac{3p\Phi_f}{2j}x_2 + \frac{f}{j}x_3 + \frac{1}{j}T_L
\]  

(6)

b). step2. Choose the following candidate Lyapunov function:

\[
V_1 = \frac{1}{2}e_{x_3}^2
\]  

(7)
Its derivative is given by:

\[ \dot{V}_1 = e_{x_3} \dot{x}_3 = e_{x_3} \left( x_{3\text{ref}} - \frac{3\Phi_f}{2j} x_2 + f \frac{1}{j} x_3 + \frac{1}{T_L} \right) \quad (8) \]

The backstepping design method considers the d-q axes currents \( x_1 \) and \( x_2 \) as the virtual control elements and specify its desired behavior, which are called stabilizing function in the backstepping design terminology as follows:

\[
\begin{cases}
  x_{1\text{ref}} = 0 \\
  x_{2\text{ref}} = \frac{2}{3\Phi_f} (f x_3 + T_L + j K_{x_3} e_{x_3})
\end{cases} \quad (9)
\]

With \( K_{x_3} \) is a positive constant.

According to Lyapunov stability theory, in order to guarantee the reference tracking, the condition \( \dot{V}_1 < 0 \) must be verified.

Substituting (9) in (8) the derivative of \( V_1 \):

\[ \dot{V}_1 = -K_{x_3} e_{x_3}^2, K_{x_3} > 0 \quad (10) \]

Therefore, the speed error approaches zero and global asymptotic stability is achieved.

2.4. Backstepping Current Controller:

The asymptotic stability of the origin of the system (1) is satisfied. The following current errors are defined:

\[
\begin{aligned}
  e_{x_1} &= x_{1\text{ref}} - x_1 \\
  e_{x_2} &= x_{2\text{ref}} - x_2 \quad \text{with } x_{1\text{ref}} = 0
\end{aligned} \quad (11) \]

Their dynamics can be written:

\[
\begin{aligned}
  \dot{e}_{x_1} &= \dot{x}_{1\text{ref}} - \dot{x}_1 = \frac{R_s}{L} x_1 - p x_2 x_3 - \frac{1}{L} V_d \\
  \dot{e}_{x_2} &= \dot{x}_{2\text{ref}} - \dot{x}_2 = \frac{2}{3\Phi_f} (f x_3 + T_L + j K_{x_3} e_{x_3}) + \frac{R_s}{L} x_2 + p x_1 x_3 + \frac{\Phi_f}{L} x_3
\end{aligned} \quad (12)
\]

To analyze the stability of this system the following Lyapunov function is proposed:

\[ V_2 = \frac{1}{2} (e_{x_3}^2 + e_{x_1}^2 + e_{x_2}^2) \quad (13) \]

In this section, its derivative along the trajectories (6), (12) and (13) is:

\[
\begin{aligned}
  \dot{V}_2 = e_{x_3} \dot{e}_{x_3} + e_{x_1} \dot{e}_{x_1} + e_{x_2} \dot{e}_{x_2} = -k_{x_3} e_{x_3}^2 - k_{x_1} e_{x_1}^2 - k_{x_2} e_{x_2}^2 + e_{x_1} [k_{x_1} e_{x_1} - \frac{V_d}{L} + \frac{R_s}{L} - x_3 x_2] + e_{x_1} [k_{x_2} e_{x_2} + \frac{2(k_{x_2} f - \phi)}{3\Phi_f} (e_{x_2} - k_{x_3} e_{x_3}) + \frac{3\Phi_f}{2j} e_{x_3} - \frac{R_s}{L} - x_3 x_1 + x_3 x_1
\end{aligned} \quad (15)
\]

The expression (15) requires the following control laws:

\[
\begin{aligned}
  V_d &= k_{x_1} L e_{x_1} + R_s x_1 - L x_3 x_2 \\
  V_q &= \frac{k_{x_2} (k_{x_2} f - \phi)}{3\Phi_f} e_{x_2} - k_{x_3} e_{x_3} \frac{R_s}{L} + L x_3 x_1 + x_3 \Phi_f
\end{aligned} \quad (16)
\]

With this choice, the derivatives of (14) become:

\[ \dot{V}_2 = -k_{x_1} e_{x_1} - k_{x_2} e_{x_2} - k_{x_3} e_{x_3} < 0 \quad (18) \]

This means that the tracking error will converge asymptotically to zero.

2.5. Speed and Position Observer Design using Extended

In this section, the EKF observer will be applied to estimate the PMSM rotor speed and position in order to achieve the feedback process control performed by the previous backstepping control strategy.
2.6. Extended Kalman Filter Principle

The EKF is a mathematical tool working in iteration and numerical way capable to reconstruct the system stats from other measurable physical variables [22]. This estimator is a predictor-corrector type that is optimal in the sense that it minimizes the estimated error covariance when some presumed conditions are met [23]. For the speed and the position estimation of PMSM, where parameters variation and measurement noise are present, EKF is the ideal one [25,26].

Algorithm

The EKF can resolve the non-linear equations with numeric iteration. This filter considers the system error effects and noises affecting measurements. So, it is more robust towards noises and perturbations.

The EKF estimation procedure is realized in two steps: prediction and correction (update).

- Prediction:
  This step gives a prediction of the state vector $\hat{x}_{d,k-1}$ at sampling time $(k)$ from the input $u_{k-1}$ and state vector $\hat{x}_{k-1|k-1}$ at previous sampling time $(k-1)$.

  First, we must:
  - Predefine the state and measurement noises covariance matrix $Q$ and $R$.
  - Initialize the filter error covariance matrix $P$.

  After the predicted state estimate and covariance are calculated as:

  $\begin{align*}
  \mathbf{P}_{d,k-1} &= \mathbf{F}_{k}\mathbf{P}_{k-1|k-1}\mathbf{F}_{k}^{T} + \mathbf{Q}_{k-1} \\
  \hat{\mathbf{x}}_{d,k-1} &= \mathbf{F}_{k}\hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_{k}\mathbf{u}_{k-1}
  \end{align*}$

  Where

  $\mathbf{F}_{k} = \frac{\partial f(\hat{\mathbf{x}}_{k-1|k-1},\mathbf{u}_{k-1})}{\partial \hat{\mathbf{x}}_{k-1}} |_{\hat{\mathbf{x}}_{k-1} = \hat{\mathbf{x}}_{d,k-1}}$;

  $\mathbf{H}_{k} = \frac{\partial h(\mathbf{x}_{k})}{\partial \mathbf{x}_{k}^{T}} |_{\mathbf{x}_{k} = \hat{\mathbf{x}}_{d,k-1}}$

- Update (correction):
  In this step the updated states estimate $\hat{\mathbf{x}}_{d,k}$ are obtained from the predicted estimated states $\hat{\mathbf{x}}_{d,k-1}$ by adding a correction term $K_{k}\hat{\mathbf{y}}_{k}$ to the predicted value.

  Where $K_{k}$ is the Kalman gain.

  The EKF gain matrix is calculated by the expression:

  $\mathbf{K}_{k} = \mathbf{P}_{d,k-1}\mathbf{H}_{k}^{T}(\mathbf{H}_{k}\mathbf{P}_{d,k-1}\mathbf{H}_{k}^{T} + \mathbf{R}_{k})^{-1}$

  The updated state estimate given at the output of the observer is determined by:

  $\hat{\mathbf{x}}_{d,k} = \hat{\mathbf{x}}_{d,k-1} + K_{k}\hat{\mathbf{y}}_{k}$

  where

  $\hat{\mathbf{y}}_{k} = \mathbf{y}_{d,k} - \hat{\mathbf{y}}_{d,k-1}$

  $\mathbf{y}_{d,k} = h(\hat{\mathbf{x}}_{k})$: Actual output vector

  $\hat{\mathbf{y}}_{d,k-1} = h(\hat{\mathbf{x}}_{d,k-1})$: Predicted output vector

  Here we define the filter error updated covariance

  $\mathbf{P}_{d,k} = (I - K_{k}\mathbf{H}_{k})\mathbf{P}_{d,k-1}$

  The EKF reposes on some hypothesis especially the noise. Effectively, it supposes that the noises with affect the model are Gaussian and white and are decorrelated from estimated states [19].

  **PMSM Rotor Speed and Position Estimation using Extended Kalman Filter**

  The PMSM model with state and measurement noises is:

  $\begin{align*}
  \frac{dx}{dt} &= f(x) + g.u + w \\
  y &= h(x) + v
  \end{align*}$
Moreover:
Note that

Where:

The state vector that we want to estimate, given as follow:

\[ x_k = [i_d, i_q, \omega, \theta]^T = [x_1, x_2, x_3, x_4]^T \]

\[ y_k = [h, h]^T \]

\[ u_k \]: The Control law;
\[ w_k \]: Process noise;
\[ v_k \]: Measurement noise;
\[ w_k \] and \[ v_k \] are random disturbances. The noise covariance matrixes are defined as follows:

\[ Q = \text{cov}(w) = \text{cov}(w, w^T) = E(w, w^T) \]
\[ R = \text{cov}(v) = \text{cov}(v, v^T) = E(v, v^T) \]

\[ f(x_{k-1}, u_{k-1}), h(x_k) \]: Nonlinear functions, where:

\[ f(x_{k-1}, u_{k-1}) = x_{k-1} + \dot{x}_{k-1}T_s \quad \text{and} \quad h(x_k) = \left[ \begin{array}{c} i_d_k \\ i_q_k \end{array} \right] \]

Note that \( T_s \) is the sampling time.

Moreover:

\[ f(x_{k-1}, u_{k-1}) = \begin{bmatrix} (1 - T_s \frac{R_s}{L}) x_{1,k-1} + T_s p x_{3,k-1} x_{2,k-1} + T_s \frac{1}{L} V_{d,k-1} \\ (1 - T_s \frac{R_s}{L}) x_{2,k-1} - T_s p x_{3,k-1} x_{1,k-1} - T_s \frac{1}{L} p \Phi x_{3,k-1} + T_s \frac{1}{L} V_{q,k-1} \\ (1 - T_s \frac{f}{J}) x_{3,k-1} + T_s \frac{3p}{2J} \Phi x_{2,k-1} - T_s \frac{1}{J} T_\Phi x_{4,k-1} \\ x_{4,k-1} \end{bmatrix} \]

\[ h(x_k) = \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} \]

So,

\[ F_k = \begin{bmatrix} 1 - T_s \frac{R_s}{L} & T_s p x_{3,k} & T_s p x_{2,k} & 0 \\ 1 - T_s \frac{R_s}{L} & 1 - T_s \frac{R_s}{L} & -T_s p x_{3,k} - T_s \frac{1}{L} p \Phi_f & 0 \\ 0 & \frac{T_s}{2J} \Phi_f & 1 - T_s \frac{f}{J} & 1 \end{bmatrix} \]

\[ H_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \]

## 3. SIMULATION RESULTS

In order to validate and verify the proposed sensorless nonlinear controller, digital simulations are performed in Simulink-MATLAB with regards to the following tasks: possibility speed changing and load torque disturbance. The used bloc scheme is described in Fig.1.

The nominal PMSM parameters are shown in table 1. Considered the parameter of algorithm controller in simulation as: \( k_\Omega = 700 \); \( k_d = 10000 \); \( k_q = 10000 \).

The Fig.2. shows that the response of the dynamic system worked in parameters nominal and we applied all the possibility (Low speed-high speed and in the sense inverse) so as:

In [0.5s-1.5s] we inject the torque load in 0.7s at 1.3s for 0.8Nm and we can see in low speed this command rejected that, also in high speed we reload another torque load (10Nm) in [2.2s-2.9s] then the Fig2 (a) show the response of the system is respect rapidly the reference in 0.07s of trajectory in [2s-3s] with small error.

About the torque and the flux are goodness response as shown in the Fig 2(f and i) that ensure the stabilization and the estimator of the rotor speed is aligned and can truck very well trajectory of the reference speed with a small dip appears the Fig2 (a) at the instant of the load torque at 2.2s.

The results simulation proved the trust and satisfactory performance of the proposed scheme control at dynamic high-low speed operation.
Table 1 The PMSM Parameters

<table>
<thead>
<tr>
<th>Table Head</th>
<th>The PMSM parameters</th>
<th>Designation</th>
<th>Notation</th>
<th>Rating values</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Stator resistor</td>
<td>$R_s$</td>
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<td></td>
</tr>
<tr>
<td>2</td>
<td>Stator inductance</td>
<td>$L_{d},L_{q}$</td>
<td>0.0058 H</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Number of poles</td>
<td>$p$</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Rotor magnet flux</td>
<td>$\Phi_f$</td>
<td>0.1546 Wb</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Moment of inertia</td>
<td>$j$</td>
<td>0.00176 kg.m$^2$</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Friction coefficient</td>
<td>$f$</td>
<td>0.000388 N.m</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1 Scheme block diagram of proposed sensorless nonlinear controller of PMSM
(a) Speed estimator tracking

(b) The speed tracking error

(c) Dynamic speed trajectory tracking
Fig. 3. Simulated results of proposed sensorless nonlinear controller of the PMSM.
4. CONCLUSIONS

In this paper, a backstepping control scheme combined with Extended Kalman Filter to control and estimate speed tracking of PMSM is illustrated. This work offers a choice of design tools to accommodate uncertainties and nonlinearities. This study has successfully demonstrated the design of the backstepping control for the speed control of a permanent magnet synchronous motor. The performance of the proposed controller has been investigated in simulation using MATLAB Simulink environments. The different results show its effectiveness and robustness at tracking a reference speed.

References


