A New Design Method for Fractional Order Proportional Integral (FO-PI) Controller of 3x3 Multivariable Systems

Sami LAIFA ^{(1)*}, Badreddine BOUDJEHEM ⁽²⁾

⁽¹⁾ Advanced Control Laboratory, Department of Electronics and Telecommunications, Université 8 Mai 1945 Guelma BP 401, Guelma 24000, Algeria

⁽²⁾ Advanced Control Laboratory, Department of Electrotechnics and Automatic, Université 8 Mai 1945 Guelma BP 401, Guelma 24000, Algeria
[*samilaifa@yahoo.fr](mailto:samilaifa@yahoo.fr) ; laifa.sami@univ-guelma.dz

Abstract: This paper presents a new design method of Fractional Order Proportional Integral Controller (FO-PI) for 3x3 multivariable system (three-input-three-output). The Optimal parameters of the FO-PI controllers are tuning by minimizing performance index criterion as objective function. The irrational transfer function of the fractional operator is performed by means of diffusive representation and allows to formulate the optimization problem as a function of fractional order. The simulation results show that the performance of the response obtained by diffusive approach -based FO-PI are better than whose obtained by the classical controllers.

Keywords: Fractional order (FO-PI) Controller; 3x3 Multivariable System; Diffusive representation

1. INTRODUCTION

Control of multivariable systems (multi-input and multi-output (MIMO)) is generally divided into two types: decentralized control and centralized control [1-2-3-4-5]. Decentralized control is the most used in industry, given the simplicity of maintenance and implementation, as well as the performance that can be achieved by focusing on the performance of individual loops.

Several methods have been proposed to adjust the optimal parameters of the decentralized classical PI / PID controllers on multivariable systems:

[6] Presented a method based on using a sequential optimization procedure through application of the Genetic Algorithm to obtain the tuning parameters for a conventional multivariable PI controller applied to 2x2 MIMO processes.

In [7] a decentralized Conventional PI (PID) controller is obtained using the non-dimensional tuning method for TITO processes.

In [8] proposed a decentralized classical PID controller design for multivariable system (three-input-three-output(3x3)) using heuristic algorithms

The fractional order proportional integral derivative was first introduced by Podlubny in

[9] and it is considered as the generalization case of classical PID controllers. The concept of fractional calculus has been applied successfully for the control of SISO (single-input and single-output) systems:

In [10], tuning fractional-order proportional-integral-derivative controller based on minimizing performance indices criterion, where the fractional PID is achieved by using diffusive representation of fractional operator.

In [11], tuning of fractional PI controllers for fractional order system models with and without time delays.

In [12], an intelligent optimization method for designing fractional order PID controller based on Particle Swarm Optimization (PSO).

This success prompted researchers to think about introducing this concept to multivariable systems. In [13] fractional order IMC based PID (FOIMC-PID) controller design using Novel Bat optimization algorithm for TITO Process. In [14], design of decentralized fractional-order proportional-Integral-derivative(FOPID) controllers for multivariable systems. In [15], optimal Tuning method of decentralized fractional order proportional-integral (FO-PI) controller for two-input, two-output process using diffusive approach

In this work we discuss a design method for decentralized fractional PI controller based on minimizing performance indices implemented by means of diffusive approach for efficient control of the 3x3 multivariable system (three-input and three-output). The mathematical formulations of the optimization problem are posed directly to fractional parameters. The simulation results show the superiority of the diffusive approach-based FOPI on the performance of the classical controllers.

This paper is organized as follows: Section 2 illustrates the basic mathematical notions of fractional order operators and fractional order controller. Section 3 presents the principal of the proposed design method. The simulation results are given in section 4. Finally a conclusion is given in section 5.

2. FRACTIONAL ORDER OPERATORS AND CONTROLLERS

2.1. Fractional order operators:

Fractional calculus is a generalization of integration and differentiation to non-integer order operator aD_t^α given as follows [16]:

$$aD_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha} & \alpha > 0 \\ 1 & \alpha = 0 \\ \int_a^t (d\tau)^\alpha & \alpha < 0 \end{cases} \quad (1)$$

where a and t denote the limits of the operation and α denotes the fractional order $\alpha \in \mathbb{R}$ est l'ordre de l'opération.

There are various definitions of fractional differentiation-integrals available. The following definitions of fractional calculus are used widely in the area of control system [17]. A commonly used definition of fractional differo-integral is the Riemann-Liouville definition which is given by:

$${}_t D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_{t_0}^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \quad (2)$$

For $(n-1 < \alpha < n)$

and $\Gamma(\alpha)$ is Euler's Gamma function

By Grunwald-Letnikov method the α -th order differ-integration of a function $f(t)$ is defined as:

$$D_t^\alpha f(t) := \lim_{h \rightarrow 0} \frac{1}{h^\alpha} \sum_{j=0}^{\infty} (-1)^j \binom{\alpha}{j} f(t-jh) \quad (3)$$

where :

$$\binom{\alpha}{j} = \frac{\alpha!}{j! (\alpha-j)!} = \frac{\Gamma(\alpha+1)}{j! \Gamma(\alpha-j+1)} \quad (4)$$

Denotes the binomial co-efficient. The Laplace transform of Grunwald-Letnikov fractional differ-integration is:

$$\int_0^\infty e^{-st} {}_0 D_t^\alpha f(t) dt = s^\alpha F(s) \quad (5)$$

Where $F(s)$ is the normal Laplace transformation $f(t)$.

2.2. Fractional order PI^α controller:

The differential equation of the fractional order PI^α (FO-PI or PI^α) is given by:

$$u(t) = K_p \left(e(t) + \frac{1}{T_i} D^{-\alpha} e(t) \right) \quad (6)$$

where:

D is the derivative operation,

K_p is the proportional gain.

T_i is the integral time constant.

α is the integration order.

The transfer function of the fractional order PI (FO-PI) controller is given by:

$$C(s)_{fractional} = K_p \left(1 + \frac{1}{T_i s^\alpha} \right) = K_p + \frac{K_I}{s^\alpha} \\ = K_p + K_I s^{-\alpha}$$

$$\text{as: } K_I = K_p / T_i \quad (7)$$

If $\alpha = 1$, we get classical PI controller

$$C(s)_{classical} = K_p \left(1 + \frac{1}{T_i s} \right) = K_p + \frac{K_I}{s} \\ = K_p + K_I s^{-1} \quad (8)$$

2.3. Diffusive Representation:

The diffusive representation originated from the so called diffusive representation of fractional integrators, see (Montseny et al...1993). [18]. the interests of diffusive representation are to transform problems of a hereditary nature (non-rational transfers) into differential problems, which in particular allow a standard and efficient numerical approximation. The diffusive realization of the pseudo-differential operator, with impulse response h , $u \rightarrow y = H \left(\frac{d}{dt} \right) u$ is defined by the dynamic input-output system:

$$\begin{cases} \frac{\partial \psi(\xi, t)}{\partial t} = -\xi \psi(\xi, t) + u(t) \\ y(t) = \int_0^{+\infty} \mu(\xi) \psi(\xi, t) d\xi \\ \psi(\xi, 0) = 0, \xi > 0 \end{cases} \quad (9)$$

The system (9) is the diffusive realization of H . The impulse response $h(t)$ is expressed from $h(t)$ by:

$$h(t) = \int_0^{+\infty} e^{-\xi t} \mu(\xi) d\xi \quad (10)$$

The transfer function of the operator H is given by:

$$H(p) = \int_0^{+\infty} \frac{\mu(\xi)}{p + \xi} d\xi \quad (11)$$

The transfer function associated with the diffusive representation. In other words, $\mu(\xi)$ the representation is obtained directly by the inverse Laplace transformation of the impulse response $h(t)$. The diffusive representation is therefore a "counter-sense" use of the Laplace transform:

$$\mu(\xi) \xrightarrow{\mathcal{L}_\xi} h(t) \xrightarrow{\mathcal{L}_t} H(p), \quad \mu(\xi) = \mathcal{L}^{-1}(h(t)) \quad (12)$$

For example the diffusive symbol of the fractional integrator s of order α :

$$H\left(\frac{\partial}{\partial t}\right) \text{ où } H(p) = \frac{1}{p^\alpha} \text{ for } 0 < \alpha < 1 \quad (13)$$

The diffusive symbol is expressed as (see laudebat et al. (2004)) [19]:

$$\mu(\xi) = \xi^{-\alpha} \frac{\sin(\alpha\pi)}{\pi} \quad \xi > 0$$

where α is the order of integration.

and The diffusive realization of the controller PI^α with input u and output y may be expressed by [15]:

$$\begin{cases} \partial_t \psi(\xi, t) = -\xi \psi(\xi, t) + u(t) \\ y(t) = \int_0^{+\infty} v(\xi) \psi(\xi, t) d\xi \end{cases} \quad (14)$$

the transfer function of the fractional controller PI^α by means of diffusive representation are:

$$C(s) = \int_{-\infty}^{+\infty} \frac{v(\xi)}{p + \xi} d\xi \quad (15)$$

Where v the diffusive representation know as follows:

$$v(\xi) = K_p \left(\delta(\xi) + \frac{1}{T_i} \frac{\sin(\alpha\pi)}{\pi} \xi^{-\alpha} \right) \quad (16)$$

3. PROPOSED DESIGN METHOD

Consider a multivariable process with three - input and three-output given by figure (1)

Fig. 1. Multivariable process with three-input and three-output

The 3x3 system is defined by the transfer function as a form

$$F(s) = \begin{bmatrix} F_{11}(s) & F_{12}(s) & F_{13}(s) \\ F_{21}(s) & F_{22}(s) & F_{23}(s) \\ F_{31}(s) & F_{32}(s) & F_{33}(s) \end{bmatrix} \quad (17)$$

The decentralized fractional PI controller ($FO - PI_{1,2,3}$) is described by equation (18)

$$G_c(s) = \begin{bmatrix} G_{c1}(s) & 0 & 0 \\ 0 & G_{c2}(s) & 0 \\ 0 & 0 & G_{c3}(s) \end{bmatrix} = \begin{bmatrix} FO - PI_1 & 0 & 0 \\ 0 & FO - PI_2 & 0 \\ 0 & 0 & FO - PI_3 \end{bmatrix} \quad (18)$$

A 3x3 Multivariable system with three decentralised fractional PI controllers is shown in Figure 2.

Fig. 2 A 3x3 Multivariable system with three decentralised fractional PI controllers.

Thus the transfer function of the three fractional PI^{a_1}, PI^{a_2} and PI^{a_3} controller by means of diffusive Approach are:

$$FO - PI_1 = \int_{-\infty}^{+\infty} \frac{v_1(\xi)}{p + \xi} d\xi \quad (19)$$

$$FO - PI_2 = \int_{-\infty}^{+\infty} \frac{v_2(\xi)}{p + \xi} d\xi \quad (20)$$

$$FO - PI_3 = \int_{-\infty}^{+\infty} \frac{v_3(\xi)}{p + \xi} d\xi \quad (21)$$

Our objectives are to tuning the nine parameters ($K_{p1}, K_{i1}, K_{p2}, K_{i2}, K_{p3}, K_{i3}, \alpha_1, \alpha_2, \text{ and } \alpha_3$) of the three decentralized fractional PI controller that minimizing the performance index error defined by the objective function

$$\min J(K_{p1}, K_{i1}, K_{p2}, K_{i2}, K_{p3}, K_{i3}, \alpha_1, \alpha_2, \alpha_3) = \int_0^{\infty} f(e(t)) dt \quad (22)$$

The steps to determine the three fractional PI controllers parameters can be summarized as follows:

- Implement the feedback control system in Matlab/Simulink including diffusive approximation of the three fractional PI controllers through Simulink model
- Initial parameters for the controller are determined using Ziegler-Nichols based on the stability margin.
- Use a function of Matlab optimization toolbox to minimize the objective function J .

4. SIMULATION AND RESULTS

As a simulation studie , we consider a 3x3 Multivariable process given by [20-21],

$$F(s) = \begin{bmatrix} \frac{119e^{-5s}}{21.7s + 1} & \frac{153e^{-5s}}{337s + 1} & \frac{-2.1e^{-5s}}{10s + 1} \\ \frac{37.0e^{-5s}}{500s + 1} & \frac{76.7e^{-5s}}{28s + 1} & \frac{-5e^{-5s}}{10s + 1} \\ \frac{93e^{-5s}}{500s + 1} & \frac{-66.7e^{-5s}}{166s + 1} & \frac{-103.3e^{-5s}}{23s + 1} \end{bmatrix} \quad (23)$$

The optimized parameters of fractional and classical controllers (FO-PI and C-PI) obtained by the proposed method are

organized in Table 1 and Table 2 respectively.

Parameters			
	$K_{P1,2,3}$	$K_{I1,2,3}$	$\alpha_{1,2,3}$
$FO - PI_1$	0.0195	5.153e-005	1.0307
$FO - PI_2$	0.0354	6.3753e-005	1.0304
$FO - PI_3$	-0.028	-6.534e-005	1.3775

Table 1 Optimized parameters of decentralized FO-PI controllers

Table 2 Optimized parameters of decentralized FO-PI controllers

Parameters		
	$K_{P1,2,3}$	$K_{I1,2,3}$
$C - PI_1$	0.0231	0.001649
$C - PI_2$	0.0335	0.002311
$C - PI_3$	-0.0310	-0.0024934

Fig. 3, shows the step response of the first, second, and third outputs ($Y1, Y2$ and $Y3$) of the controlled system with decentralized fractional and classical controllers.

Through the responses ($Y1, Y2, \text{ and } Y3$) it is clear to us that the performance of fractional PI controllers is better than classical controllers in terms of Overshoot and settling time.

From the simulation results it can be seen that the proposed controller gives good performance and satisfactory results.

The work presented in this paper and the representative in the design of fractional controllers based on diffusive representation for multivariable systems gave great success to him despite the difficulty of implementation.

5. CONCLUSION

In this work, a new design method of Fractional Order Proportional Integral (FO-PI) controller was implemented with the aim of improving the fractional control for complex 3x3 multivariable systems. The optimum controllers parameters (fractional and classical) for multivariable system using MATLAB optimization toolbox is addressed based on minimizing the ISE performance index criterion (Integral square error). Diffusive representation was used to make an irrational transfer function of the fractional operator to a standard transfer function of the

decentralized FO-PI controller. The simulation results show that, the responses with fractional PI controller have a minimum overshoot and settling time in comparison with classical PI controller. Therefore, the performance of the fractional PI controller based on the diffusive approach is clearly superior to the performance of classical PI controllers.

Fig. 3 Output responses for 3x3 system. (a) System output y_1 . (b) System output y_2 and (c) System output y_3 with set point changes at $t=0$, $t=100$, and $t=200$.

References

- [1] D. Hajare and B. M. Patre, Decentralized PID controller for TITO systems using characteristic ratio assignment with an experimental application, *ISA Transaction*, 2015, 59, 385-397.
- [2] K. Maghade, B. M. Patre. Decentralized PI/PID controllers based on gain and phase margin specifications for TITO processes. *ISA Transactions*, vol. 51, no. 4, pp. 550-558, 2012.
- [3] Q Xiong, Cai WJ. Effective transfer function method for decentralized control system design of multi-input multi-output processes. *J Process Control* 16(8):773-784, (2006).
- [4] X Luan, Q Chen, F Liu. Centralized PI control for high dimensional multi-variable systems based on equivalent transfer function. *ISA Trans* 2014;53 (4):15561
- [5] VV Kumar, Rao VSR, M Chidambaram. Centralized PI controllers for interacting multivariable processes by synthesis method. *ISA Trans* 2012;51(3):4009.
- [6] E Turco Neto, A Syed. Imtiaz, S Ahmed. A sequential optimization procedure to tune MIMO PID controllers. *Indian Control Conference (ICC)*, January 4-6, 2017.
- [7] S. Tavakoli. I. Griffin. P. J. Flemina. Tuning of decentralised PI (PID) controllers for TITO processes. *Control Engineering Practice*. vol. 14, no. 9, pp. 1069-1080, 2006. DOI: 10.1016/j.conengprac.2005.06.006.
- [8] H. Devikumari, V. Vijayan Decentralized PID Controller Design for 3x3 Multivariable System using Heuristic Algorithms *Indian Journal of Science and Technology*, Vol 8(15), DOI: 10.17485/ijst/2015/v8i15/70394, July 2015
- [9] Podlubny, "Fractional order systems and $PI^{\lambda}D^{\mu}$ controllers," *IEEE Transactions on Automatic Control*, vol. 44, 208-214, 1999
- [10] B. Boudjehem, and D. Boudjehem. "Fractional PID Controller Design Based on Minimizing Performance indices." *IFAC-PapersOnLine* 49.9 (2016): 164-168
- [11] Narang, S. L. Shah, and T. Chen, "Tuning of fractional PI controllers for fractional order system models with and without time delays," in *American Control Conference*, 2010, pp. 6674-6679.
- [12] J. Cao. and, B. Cao. Design of fractional order controller based on particle swarm optimisation. *International journal of Control Automation, and system s*, 4,775-781. (2006).
- [13] S.K.Lakshmanaprabu,USaburaBanu,P.R.He mavathy.Fractional order IMC based PID controller design using Novel Bat optimization algorithm for TITO Process.*Energy Procedia*,vol.117,pp.1125- 1133,2017.
- [14] K. Bettou and A. Charef.,Design of decentralized fractional order $PI^{\alpha}D^{\beta}$ controllers for mimo systems , *International Conference on Automatic control, Telecommunications and Signals (ICATS15)*. November 16-18, 2015
- [15] B. Boudjehem S. Laifa, Optimal Tuning Method of decentralised Fractional PI Controller for TITO Process using Diffusive

- Approach. Proceedings of the International Conference on Recent Advances in Electrical Systems, Tunisia, 2018
- [16] Poudlubny, Fractional Differential Equations, Academic Press, San Diego, USA, 1999.
- [17] Oldham K, Spanier J. The fractional calculus : theory and applications of differentiation and integration to arbitrary order. Elsevier; 1974. ISBN 0486450015. Article: Review of fractional PID controller
- [18] Montseny, G. (2004). Simple approach to approximation and dynamical realization of pseudo differential time operator s such as fractional ones. IEEE Transactions on Circuits and Systems II, 51, 613–618.
- [19] Laudebat, L., Bidan, P., and Montseny, G. (2004). Modeling and optimal identification of pseudo differential dynamics by means of diffusive representation part I: modeling. IEEE Transactions on Circuits and Systems, 51, 1801–1813.
- [20] A.P.Loh,V.U.Vasnani, Describing function matrix for multivariable systems and use in Multiloop PI design, Journal of process control 4(1994) 115-120.
- [21] R.Hanuma Naik, D.V.Ashok Kumar and K.S.R Anjaneyulu "Control Configuration Selection and Controller Design for Multivariable Processes Using Normalized Gain.", International Journal of Electrical, Computer, Electronics and Communication Engineering Vol:8, No:10, 2014.